Ability Grouping by Multiple Discriminant Analysis

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Christensen (3), Ivanoff (6), Calia (1), Hall (5), Sprinthall (1), and Kelly, Veldman and McGuire (7) have all utilized discriminant functions for classification of students and pupils at the college and high school level. Most classification has centered on prediction of vocational choice or school dropout. Little if any use has been made of discriminant analysis to classify subjects into groups for instructional purposes.

The purpose of this study was to evaluate a technique by which pupils might be organized into homogeneous instructional groups in English. The technique used involved the placement of pupils in sections defined in terms of the emphasis given to grammar, expression and reading in typical ninth or tenth grade English courses.

Pupils were classified into one of the four following groups based on data from the prior school-year records:

a. Pupils who show a relatively greater deficiency in English grammar.

b. Pupils who show a relatively greater deficiency in English expression.

c. Pupils who show a relatively greater deficiency in reading English.

d. Pupils who show no one single deficiency to a greater degree than others (or who are relatively equal in achievement in reading, expression and grammar).

The criterion for correct classification was the placement assigned by the pupil's current English teacher. This criterion classification was based on observational evidence gathered during one semester.

The essential problem of this study was to evaluate the accuracy with which classification into the above mentioned groups could be predicted using data accumulated during the year prior to assignment. Classifications were made using functions obtained through multiple discriminant analysis.

Procedures

Four samples, one from each of four school districts, were obtained for empirical investigation of the method. In each school sample, one half of the sample was used to obtain discriminant functions and the other half was used as a cross-validation group. The four pupil samples obtained were:

1. Two hundred ten ninth grade pupils from Bettendorf Senior High School, Bettendorf, Iowa.

2. One hundred fifty ninth grade pupils from Peet Junior High School, Cedar Falls, Iowa.

3. One hundred thirty-four ninth grade pupils from Central Junior High School, Iowa City, Iowa.

4. One hundred twelve tenth grade pupils from Grinnel Community High School, Grinnell, Iowa.

Data collected on the ninth grade samples included some or all of the following:

1. Selected scores of the Iowa Tests of Basic Skills, i.e., Vocabulary (V), Reading Comprehension (R), Spelling (L-1), Capitalization (L-2), Punctuation (L-3), Usage (L-4), Language Total (LT), Map Reading (W-1), Reading Graphs and Tables (W-2), Use of Reference Materials (W-3), Work-Study Total (WT), Arithmetic Concepts (A-1), Arithmetic Problem Solving (A-2), Total Arithmetic (AT), and the Composite (C).

2. Sex (boys = 1, girls = 2).

3. Mark-Point Average.

4. Second semester English marks, eighth grade.

5. Responses to the following questions from CardPac (CardPac, 1965):

   Question 9 - Which of the following statements best describe the highest level of Education which your father has attained?

   Question 10 - Which one of the following statements best describe the highest level of Education which your mother has attained?

Question 28 - Which of the following statements best describes your feeling about school this year?

Question 40 - Realistically, considering your abilities, your financial resources, family problems, etc., what is the highest level of education you expect to attain?

6. Teacher deficiency classification (criterion measure).

Data available from the tenth grade sample was the same as those above except that the ten scores of the Iowa Tests of Educational Development were used instead of the ITBS. The ITED scores were Social Studies Background (ITED-1), Natural Sciences Background (ITED-2), Written Expression (ITED-3), Quantitative Thinking (ITED-4), Interpretation-Social Studies (ITED-5), Interpretation-Natural Sciences (ITED-6), Interpretation-Literature (ITED-7), General Vocabulary (ITED-8), Composite on 1-8 (ITED-C), and Use of Sources of Information (ITED-9). Mark-point average, English marks and CardPac responses were those obtained during the ninth grade.

Analysis of the data consisted of five major steps. These were:

1. An analysis of variance among and within each of the four deficiency groups on each of the predicting variables.

2. A multiple discriminant function analysis using the procedures described by Cooley and Lohnes (4) and computed on an IBM 7044 computer.

3. Location of the centroids of the criterion groups in the discriminant space.

4. Testing for the significance of discrimination among groups in the original test space by procedures developed by Rao (10) and illustrated by Ottman, Ferguson and Kaufman (9). These procedures employ a statistic originally developed by Mahalanobis and described by Ottman, et al (9).

5. Computation of chi-squares to test the hypothesis of chance classification of the cross-validation pupils. These pupils were classified using discriminant functions obtained in (2) above. Probabilities of group membership constituted the method for classification.

Multiple discriminant analysis bears resemblance to multiple regression analysis in that a number of variables are combined into a function which maximizes a fit of a line or plane to data points. However, multiple regression functions are used to predict a deviation from a mean score on a criterion measure whereas discriminant functions are used to classify subjects into criterion groups. These groups are defined by the degree to which the means and dispersions of data points for the groups differ in the test space. Thus, unlike multiple regression, several functions may be developed which describe the location of group centroids in either the original test space or a transformation of the test space, the discriminant space.

Results

Analyses of variance among the deficiency group means within each school sample yielded F-ratios significant at the .05 level for the following variables:

(a) Bettendorf sample - R, LT, WT, AT, CO-OP, ENGLISH-8, SPEECH-8, and HISTORY-8.

(b) Bettendorf sample - V, R, LT, WT, MPA, ENGLISH-8.

(c) Iowa City sample - V, R, LT, AT, CP-28, CP-40, MPA, and ENGLISH-8.

(d) Grinnell sample - none.

Only the tenth grade sample failed to yield significant differences among the group means on at least one variable. In two samples, the Arithmetic total of the ITBS was significant despite the fact that the test is designed to measure Arithmetic not English achievement. In veiw of the high correlations usually found among various course achievement measures this finding is not surprising.

Tables 1-4 present the results of the discriminant analyses. The discriminant analyses yielded normalized and scaled eigenvectors and associated eigenvalues characteristic of the matrices used in the analyses. The scaled vectors indicate those variables contributing the greatest amount to discrimination between groups as judged from the absolute magnitude of the coefficients. As would be expected from the results of the analyses of variance, the analysis of the tenth grade sample failed to yield a significant discrimination among groups. This conclusion is based on the F-ratio derived from the Wilkes Lambda coefficient computed in the analysis.

TABLE 1

Roots and Discriminant Vectors Derived by the Cooley-Lohnes Procedure: First Half Bettendorf Data

Variable	Normalized Vectors			Scaled Vectors		
	I	II	III	I	II	III
1. V	-.01	.05	.02	-1.13	8.63	3.20
2. R	-.04	-.07	-.02	-6.69	-13.23	-3.73
3. LT	.11	.03	-.03	17.20	4.60	-4.74
4. WT	.02	-.07	.09	3.91	-11.75	14.76
5. AT	.07	.08	.02	11.31	12.40	3.64
6. CO-OP	.00	.10	-.04	.29	15.48	-6.83
7. SEX	.75	-.90	.76	3.75	-4.50	3.81
8. ENGLISH-8	-.34	-.01	-.39	-9.14	-.35	-10.29
9. SPEECH-8	.51	-.08	.32	12.18	-1.90	7.59
10. HISTORY-8	-.19	-.39	-.39	-5.72	-11.36	-11.54

	Latent Roots	Percent of Trace
R_I	= .449	68.38
R_{II}	= .122	18.65
R_{III}	= .085	12.97
Trace of $W^{-1}A$	= .657	Sum of R_i = .656
Wilkes Lambda	= .566	$F_{30,268}$ = 1.908 (Significant)

TABLE 2

Roots and Discriminant Vectors Derived by the Cooley-Lohnes Procedure: First Half Peet Junior High School Data

Variable	Normalized Vectors I	Normalized Vectors II	Normalized Vectors III	Scaled Vectors I	Scaled Vectors II	Scaled Vectors III
1. V	0.01	0.04	0.03	1.53	4.62	-2.84
2. R	0.11	-0.06	0.02	12.11	-6.86	2.20
3. LT	0.07	-0.01	-0.02	8.75	-0.64	-2.34
4. WT	-0.04	0.00	0.01	-5.63	0.53	1.49
5. AT	-0.02	0.00	-0.01	-2.95	0.30	-1.55
6. SEX	-0.09	0.23	0.27	-0.41	1.03	1.19
7. CP-9	0.17	0.02	0.04	3.38	0.35	0.78
8. CP-10	-0.10	0.02	0.10	-2.10	0.39	2.11
9. CP-28	-0.62	0.23	0.06	-3.38	1.25	0.34
10. CP-40	0.09	-0.09	-0.08	1.62	-1.68	-1.51
11. MPA	0.01	0.00	-0.00	3.12	1.94	-1.72
12. ENG-8	-0.74	0.94	0.95	-3.38	4.31	4.38

	Latent Roots	Percent of Trace
	R_I = .7557	61.6499
	R_{II} = .3797	30.9742
	R_{III} = .0904	7.3759
Trace of $W^{-1}A$	= 1.22583	Sum of R_i = 1.22583
Wilkes Lambda	= 0.3786	$F_{36,161}$ = 1.956 (Significant)

TABLE 3

Roots and Discriminant Vectors Derived by the Cooley-Lohnes Procedure: First Half Iowa City Data

Variable	Normalized Vectors		Scaled Vectors	
	I	II	I	II
1. V	-0.06	0.02	-7.48	2.48
2. R	0.05	-0.04	5.87	-4.52
3. LT	0.07	-0.03	8.81	-3.52
4. WT	-0.06	0.01	-7.57	0.67
5. AT	0.07	0.02	8.48	2.69
6. SEX	-0.84	0.88	-3.45	3.60
7. CP-9	0.09	0.06	2.13	1.45
8. CP-10	0.18	-0.04	4.40	-1.06
9. CP-28	-0.39	-0.19	-2.21	-1.09
10. CP-40	0.17	0.10	4.20	2.53
11. MPA	0.07	0.01	28.07	2.26
12. ENG-9	0.21	-0.42	1.33	-2.60

	Latent Roots	Percent of Trace
	R_I = 1.9507	87.0935
	R_{II} = 0.2891	12.9065
Trace of $W^{-1}A$	= 2.2398	Sum of R_i = 2.2398
Wilkes Lambda	= 0.2629	$F_{24,116} = 4.593$ (Significant)

TABLE 4

Roots and Discriminant Vectors Derived By the Cooley-Lohnes Procedure: First Half Grinnell Data

Variable	Normalized Vectors I	Normalized Vectors II	Scaled Vectors I	Scaled Vectors II
1. ITED-1	0.07	0.02	2.54	0.92
2. ITED-2	0.01	-0.24	0.27	-10.49
3. ITED-3	-0.21	0.05	-6.45	1.40
4. ITED-4	-0.06	-0.05	-2.62	-2.01
5. ITED-5	0.07	-0.15	2.78	-6.16
6. ITED-6	0.02	-0.18	0.63	-7.48
7. ITED-7	0.07	-0.03	2.58	-1.06
8. ITED-8	-0.10	-0.13	-3.32	-4.29
9. ITED-9	0.05	0.24	2.21	10.82
10. MPA	0.02	0.02	5.99	8.44
11. SEX	-0.69	0.46	-2.56	1.72
12. CP-9	0.21	-0.07	3.49	-1.21
13. CP-10	-0.10	0.45	-1.31	6.24
14. CP-28	0.17	0.58	1.20	4.01
15. CP-40	0.15	0.11	3.42	2.54
16. ENGLISH-9	-0.58	-0.20	-2.70	-0.91

	Latent Roots	Percent of Trace
	R_I = 0.4227	69.7016
	R_{II} = 0.1838	30.2984
Trace of $W^{-1}A$	= 0.607	Sum of R_i = 0.607
Wilkes Lambda	= 0.594	$F_{32, 84}$ = 0.892 (Non-significant)

Figures 1-4 present the graphical representation of the centroids for each group in the discriminant space of each analysis. The Mahalanobis D^2 statistic computed between pairs of groups in each analysis indicated the following:

(a) Significant differences between the Grammar and Balanced and between the Balanced and Reading groups in the Bettendorf sample.

(b) A significant difference between the Balanced and Expression groups of the Cedar Falls sample.

(c) A significant difference between the Reading and Grammar groups in the Iowa City sample.

(d) No differences among the groups in the Grinnell sample.

It should be pointed out that in the Iowa City and Grinnell samples, teachers failed to classify any subjects in the "Balanced" group, i.e., the group of pupils thought to show no single deficiency greater in one area than the others.

Tables 5-8 present summaries of correct and incorrect classifications resulting from application of functions (obtained on the first-half samples) for predicting the classification of pupils from the second half samples. If the functions are useful in predicting classification, a number of correct classifications beyond that expected by chance alone must be made. Only two of the four samples obtained significant classification. The chi-squares for Peet Junior High School and Central Junior High School were significant. A t-test of the hypothesis that the frequencies observed in the diagonal elements would not differ from that expected by chance was also rejected for these two samples. Approximately 52 and 71 per cent respectively were classified in the correct groups. It should be noted that with four groups chance placement would correctly place about one-fourth and one-third of the respective samples.

Conclusions

The results suggest that classification of pupils into instructional groups based on relative deficiencies is feasible for some schools. Grouping for instruction aimed at diminishing specific deficiencies appeared to be conceptually sound and the use of discriminant analysis

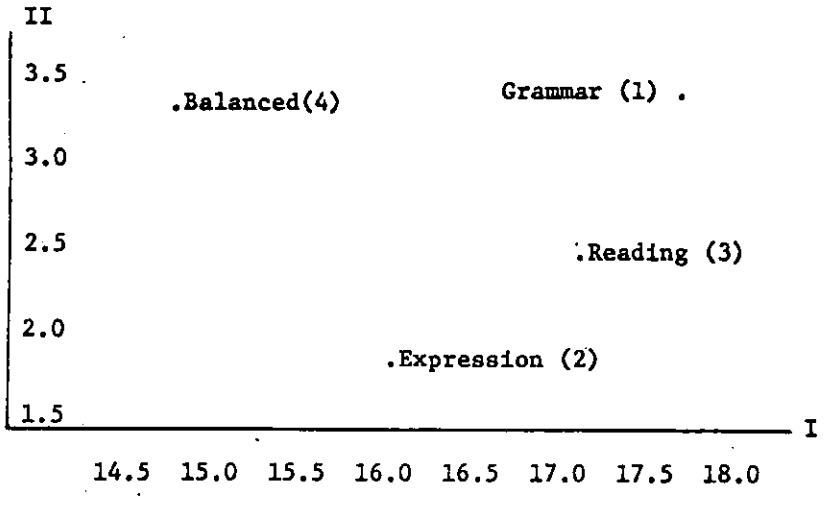

Figure 1 FIRST HALF BETTENDORF DATA

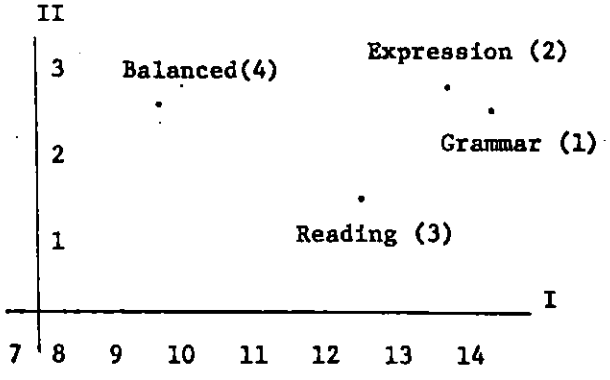

Figure 2 FIRST HALF PEET JUNIOR HIGH SCHOOL

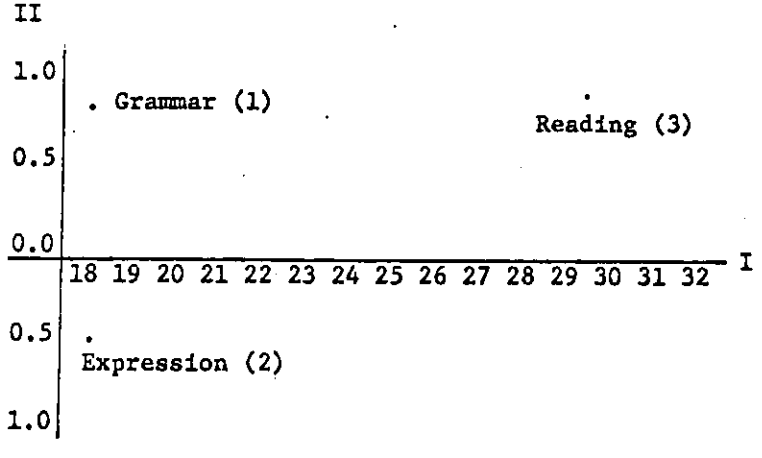

Figure 3 FIRST HALF SAMPLE IOWA CITY

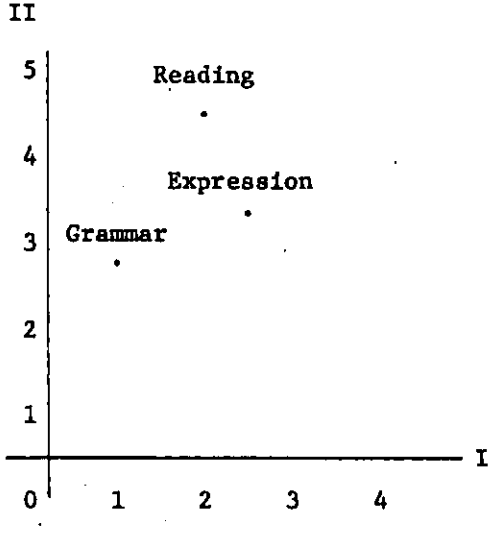

Figure 4 GRINNELL SENIOR HIGH SCHOOL DATA

CENTROIDS OF THE FOUR DEFICIENCY GROUPS IN THE SPACE OF DISCRIMINANT VECTORS I AND II

provided a technique for implementation of the principle. Further study is needed to test the technique for other subject areas. In particular, study of the reliability of the criterion, that is, teacher placement into deficiency categories, needs to be pursued.

TABLE 5

Classification Resulting for The Cross-Validation Group: Bettendorf Senior High School

True Group Placement	Predicted Group Placement 1	2	3	4	Total
1. Grammar	16	1	1	4	22
2. Expression	10	0	1	6	17
3. Reading	11	2	3	11	27
4. Balanced	13	1	4	20	38
Total	50	4	9	41	104

Chi-Square = 11.936 with degrees of freedom = 9 (non-significant)

TABLE 6

Classification Resulting for The Cross-Validation Group: Peet Junior High School

True Group Placement	Predicted Group Placement 1	2	3	4	Total
1. Grammar	0	3	0	2	5
2. Expression	7	26	1	2	36
3. Reading	0	6	0	5	11
4. Balanced	3	5	2	13	23
Total	11	40	3	22	75

Chi-Square = 25.847 Degrees of Freedom = 9 (Significant at the .05 level)

t-test computed from the diagonal values=4.59 Degrees of Freedom = 74.

TABLE 7

Classification Resulting for the Cross-Validation Group: Central Junior High School

True Group Placement	Predicted Group Placement				
	1	2	3	4	Total
1. Grammar	26	1	4	0	31
2. Expression	11	0	1	0	12
3. Reading	4	0	25	0	29
4. Balanced	0	0	0	0	0
Total	41	1	30	0	72

Chi-Square = 40.456 Degrees of Freedom = 4 (Significant)

t-test computed from the diagonal values = 5.463
Degrees of Freedom = 71

TABLE 8

Classification Resulting for the Cross-Validation Group: Grinnell Community Senior High School

True Group Placement	Predicted Group Placement				
	1	2	3	4	Total
1. Grammar	0	6	0	0	6
2. Expression	11	21	4	0	36
3. Reading	9	6	3	0	18
Total	20	33	7	0	60

Chi-Square = 8.497 Degrees of Freedom = 4 (Non-significant)

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