Florida Journal of Educational Research, 1972, 14

ARITHMETIC COMPUTATION COMPARISON IN TRADITIONAL AND INNOVATIVE SCHOOLS

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SUMMARY

In this study, arithmetic computation was the criterion variable in a comparison between an innovative and two traditional schools. The innovative school differed from the traditional schools as it placed specific emphasis on fostering independent learning by including the students in individual academic decisions.

A single-classification analysis of covariance was used to compare the three schools with arithmetic concepts as the covariate and arithmetic computation as the criterion variable. The analysis resulted in a significant difference between schools. The major contribution to the difference came from one of the traditional schools performing better in arithmetic computation than either the other traditional school or the innovative school. The traditional school that was high on computation scores was lower than the innovative and the other traditional school on the arithmetic concept measure.

INTRODUCTION

Arithmetic computation in the elementary school is examined in this study. Computation represents the segment of arithmetic that has been de-emphasized in modern arithmetic textbooks and curricula. In its place we find emphasis on concept formation and the structure of mathematics (Berkheimer, 1963; Davis, 1964; Goodlad, 1966; Mayor, 1963). The new emphasis is partially the result of the "modern" mathematics curricula that have emerged from the various innovative mathematics projects such as the University of Illinois Committee on School Mathematics, the School Mathematics Study Group (SMSG), and the Madison Project. The teaching methodology of the new curricula is toward a more individualized discovery approach rather than the traditional expository, workbook approach. Among the primary objectives in the above mentioned curricula has been the development of an understanding of mathematical concepts on the part of the student. However, there are no statements suggesting school time be made available for elementary school mathematics classes emphasizing these goals.

With a change in both the content emphasis and the method in the new arithmetic curricula, arithmetic computation has been de-emphasized. The present study compares the performance of students in modern and more traditional arithmetic curricula in an attempt to determine the effect of curricula on arithmetic computation ability.

RELATED LITERATURE

Even though these two major differences--content and method--in traditional and innovative curricula can be identified, the interpretation of data from a study where they are both independent variables is difficult because their relative contributions to a student's computational ability are unknown.

The literature reveals little definitive research in the area of arithmetic achievement as related to the general categories of instructional method or type of curriculum. Buswell (1960) concluded that it was more effective to use teacher-centered activities--textbooks and traditional teaching techniques--than to use programmed material in his comparison of two groups on a junior high school mathematics achievement test. Descriptions of both the instructional method and the source of student information were missing in this study. In a study of fifth grade students, Price, Prescott, and Hopkins (1967) concluded that classrooms with teachers who specialize in arithmetic do not have higher student arithmetic achievement compared to self-contained classrooms. The amount and type of classroom experiences that pupils were exposed to and the source material that was provided to the pupils by the teachers were not reported. Hungerman (1967) compared the mathematics achievement of fourth, fifth, and sixth grade students who studied the School Mathematics Study Group program with students of the same grade level who studied from a traditional arithmetic program. The portion of Hungerman's study that used computational skills as a criterion resulted in a significant difference in favor of the traditional program. However, the source of content material for the two groups was different and pedagogical differences of the teachers were not reported.

The above studies are representative of the current research on innovations in mathematics education to the extent that the comparisons between innovative and traditional schools involve more than one variable. Unfortunately, the unreported or uncontrolled variables of the above studies are difficult to quantify and, therefore, to remove statistically from a comparison of schools.

SAMPLE DESCRIPTION

The data from this study were collected by Jones (1965) as part of an elementary school evaluation project involving two traditional schools and an innovative school in a southeastern Florida county. The three schools were matched as closely as possible on size, socioeconomic level of parents, past student achievement, and low turnover of students. There were 109 students in the innovative school (School I), 94 students in one traditional school (School TA), and 77 students in the other traditional school (School TB). The data represent the scores of sixth grade students on the 1964 Intermediate II Stanford Achievement Test (SAT). The SAT arithmetic computation and arithmetic concepts subtests were used in the primary data analysis. The arithmetic concepts subtest is a measure of arithmetic knowledge, while the computation subtest measures ability to add, subtract, multiply, and divide. All tests were timed tests; the authors term the allotted time as adequate. The arithmetic subtests of the SAT are judged to be a satisfactory measure of arithmetic achievement and received the following evaluation by Bryan: "In providing a measure of that phase of the traditional mathematics curriculum known by the general term 'arithmetic', the 1964 Stanford Achievement Test continues to be outstanding among tests of its kind." (1965, pp 909-910).

The county textbook purchasing system provided the same source of content information to all students in the sample. The county also suggests a sequence of textbook presentation to the teachers. The textbook used by all teachers in the sample was Elementary School Mathematics, published by Addison-Wesley.

The traditional schools represent self-contained classrooms with one teacher per class, an assigned desk for each pupil, and a study sequence such that all pupils spend the same amount of time in each subject covered during the school day. The innovative school has a more individualized program with two or more teachers in a large room, different topics being studied simultaneously, and daily pupil-teacher meetings to prepare a study list which informs the student of how much time he should spend studying in each content area. Pupils are permitted to work individually, in small groups with the teacher, and in small groups without the teacher. In addition to the content goal, students in the innovative school are expected to assume a major role in deciding their type of study, rate of progress, and readiness for the succeeding topic.

The data from the two traditional schools were not pooled because a difference in their attitude toward the research project was perceived. School TA was more enthusiastic about cooperating in the research project than was School TB and it was thought that whatever factors caused this difference might also manifest themselves on the variables being analyzed in this study.

PROCEDURE

The data were analyzed by a multiple linear regression technique that used the arithmetic concept scores as a covariate. The covariate, a measure of knowledge of general arithmetic concepts, was used to account for any systematic differences in arithmetic knowledge that may have existed between the three schools in the sample. Arithmetic concepts was considered the most relevant variable to compensate for any arithmetic ability difference between students of the three schools.

A complicating factor in this design is that the covariate as well as the criterion variable can be affected by the school that the student attends. Bottenberg and Ward (1963) describe a statistical design for this condition that allows the development of a full and a restricted regression model by assuming that the three schools have a constant effect on the pupil's arithmetic concept knowledge.

A precondition to interpretation of the results of an analysis of covariance is that the regression lines of the groups being compared are homogeneous (parallel). Failure to pass the test of homogeneity of regression means that one of the treatment groups differs significantly from the others on the relationship between the covariate and the criterion. A difference between groups from an analysis of covariance with non-parallel regression lines could be caused by a differential effect of the covariate rather than by the effect of the different treatments.

The homogeneity of regression test determines if the amount of change in arithmetic computation score per unit of arithmetic concept score is the same for all treatment groups over the observed range of arithmetic concept scores. A multiple linear regression approach can be used to make this test by defining appropriate full and restricted regression models. The full model utilized the arithmetic concept scores for each of the three schools as predictors of arithmetic computation by computing partial regression weights which reflect the increase in arithmetic computation for a unit increase in arithmetic concepts for all three groups

(Kelly, Beggs, and McNeil, 1967). The restricted model combines all three treatment groups as one predictor and computes one partial regression weight. The difference in the squared multiple correlation (R^2) between these two models is an indication of the amount of difference in the relationship between arithmetic computation and arithmetic concepts for the three treatment groups.

Three null hypotheses are tested:

Hypothesis I: There is no difference between the arithmetic computation scores of boys and girls within each school.

Hypothesis II: The size of the unit in arithmetic computation scores associated with a unit of the pupil's arithmetic concept score is the same for Schools I, TA, and TB.

If Hypothesis II is accepted, it is legitimate to test the third hypothesis:

Hypothesis III: There is no difference between the arithmetic computation scores of Schools I, TA, and TB over the observed range of arithmetic concept scores.

RESULTS

A t-ratio for differences between means resulted in probability values of .51, .57, and .89 in Hypothesis I for Schools TB, I, and TA respectively. Hypothesis I was, therefore, accepted and the remaining analyses were performed on the pooled scores for the boys and girls within each school.

The tests for homogeneity of regression yielded a probability of 57, so Hypothesis II was accepted. This result allows testing of the third hypothesis and shows that none of the three schools exhibited differential computation skills for either the upper or lower levels of concept knowledge.

The computation of the F-ratio for Hypothesis III was done by defining full and restricted multiple linear regression models for the analysis of treatment effects when covariates are influenced by treatments (Bottenberg and Ward, 1963). The squared multiple correlation (R^2) difference between the two models was .027, with an F-ratio of 7.1 and an associated probability of .001. Hypothesis III was, therefore, rejected.

With a significant overall F-ratio, it is now desirable to make comparisons between the schools taken two at a time in an attempt to discover the largest contributor to the significant F-ratio. Table 1 summarizes the differences in the R^2 between the combinations of schools taken two at a time.

TABLE 1
Magnitude of the Differences between Schools for the Squared Multiple Correlations for Arithmetic Computation Test with Covariate

Schools Compared	Differences in R^2
School TB - School I	.0324
School TB - School TA	.0355
School I - School TA	.0002

Larger R^2 values occurred between School TB and School I and tween School TB and School TA than between School I and School A. The significant overall F-ratio results from one of the traitional schools being different from the other traditional school nd innovative school. A survey of the mean scores of the three chools (see Table 2) shows that School TB is higher than the other two schools on the criterion variable but lower on the covariate. There is little difference between the means of Schools I and TA on either variable.

TABLE 2

MEANS AND STANDARD DEVIATIONS OF THE ARITHMETIC COMPUTATION AND ARITHMETIC CONCEPTS SUBTESTS

		School TB	School I	School TA
Arithmetic Computation (criterion variable)	Mean	23.33	20.80	20.44
	Std. Dev	7.45	7.58	7.95
Arithmetic Concepts (covariate)	Mean	17.82	20.33	19.04
	Std. Dev	6.26	5.91	6.63

Observation of Table 2 shows that the schools lowest on the criterion measure were highest on the covariate. A stronger statement could be made to the effect that School TB has higher arithmetic computation skill if the overall F-ratio reached significance without the covariate in the analysis. The full regression model with membership to a school as predictors was compared with a restricted model with no predictive information (a single classification ANOVA analog), to give an overall F-ratio was 3.21. Had this test been done "a priori", the F-ratio would have had an associated probability of .04. The major contributions to the R^2 difference came from the same combination of schools taken two at a time that made the major contributions with the covariate. (see Table 3)

TABLE 3

MAGNITUDE OF THE DIFFERENCES BETWEEN SCHOOLS FOR THE SQUARED MULTIPLE CORRELATIONS FOR ARITHMETIC COMPUTATION WITHOUT COVARIATE

Schools Compared	Difference in R^2
School TB - School I	.0248
School TB - School TA	.0314
School I - School TA	.0005

DISCUSSION

The data analysis compared the computational ability test scores of an innovative school and two traditional schools. Results showed that one of the traditional schools tends to be different from both the other traditional school and the innovative school. Since the source materials are the same for the three schools, it is appropriate to take a closer look within the school for other factors that could account for the between-school differences.

Hunter (1967) states that the teacher is the most important single factor in a student's success in school today. Yager and Wick (1966) conclude that teacher emphasis in the classroom is an important factor in determining student learning outcomes. Schefler (1965) summarizes his study that compares a traditional, lecture-illustrative biology class with a discovery-inductive class by stating that the effects of teacher difference may be of greater significance than the effects of teaching method differences. One possibility is that the teacher emphasis in School TB was greater in the areas of speed and accuracy of computation than in the other two schools. If teacher emphasis is a dominant factor, it could also overshadow any purported differences (innovative vs. traditional) between School I and School TA. In other words, it would prevent a meaningful comparison between an innovative and a traditional school approach to arithmetic computation. The teacher emphasis referred to would not have to represent gross differences in teacher behavior. Sueltz (1953) states that drill or recurring experience is useful to gain proficiency in handling a mathematical process or procedure after it has been studied and its usefulness established. It could be that the teacher's teaching method of assigning problems is the same in both School I and School TA, but the teachers in School TB could have moved from the stage of learning the process to having their students more proficient in computation by assigning 20 problems rather than 10 as an arithmetic exercise.

The next question that should be investigated is the relationship between the arithmetic computational ability of the students and their arithmetic concept knowledge. Table 3 shows that School TB out-performed Schools I and TA on computation, even though it had lower scores on the arithmetic subtest.

An examination of the overview of seven new or innovative curricula by Goodlad (1966) shows that they do not include arithmetic computation. The primary objective of SMSG is to develop awareness of the basic properties of mathematics; the Greater Cleveland Mathematics Program was designed to help the students achieve a clear understanding of the structural interrelationship of numbers. The possibilities explored in this study and the results of the Project TALENT data analysis by Lohnes (1966) suggest that the emphasized objectives of these innovative programs are on a different intellectual dimension than the manipulative or computational skills, and that Schools I and TA place more emphasis on the objectives similar to the new curricula than does School TB.

RESULTS AND IMPLICATIONS

The present study compares the scores of sixth grade students from two traditional and one innovative school on the arithmetic computation subtest of the Stanford Achievement Test. Its purpose was to determine if the students from the innovative school, which has greater emphasis on developing independent learning by the student, would perform as well as students from the traditional schools on the computation subtest. The results indicated no significant difference on the criterion variable between schools. The largest difference was between one traditional school and the other two schools.

The need for specific information on the teacher's methods and style as part of the experimental design was demonstrated in this research and is supported by other research in curriculum comparison. It seems that the teacher can make her class very traditional within a school that has very progressive physical construction and goals by using methods consonant with her beliefs while in the classroom. The reverse can take place with a teacher in a traditional school. The need to identify the teacher behaviors relevant to a particular study would suggest the use of an observation instrument to identify specific teacher actions within the classroom.

Just as one questions the capability of a school to teach with emphasis on independent learning, concept attainment, and mathematical structures and also maximize the teaching of computation skills, it is questionable how well a school can utilize drill on computation skills and still teach the more progressive goals adequately. In the three schools in this study, the lowest in mean concept test score was highest in mean computation and vice versa. In this light, the appropriate action by a school might be to weight the contribution of a subtest of a standardized achievement test according to its relative emphasis in each school or class when making comparisons between different types of teaching programs or when evaluating the effectiveness of the school.

Because of the observed differences among the scores of the concept and computation subtest, it would be valuable to determine the extent to which concept knowledge does transfer to computation skill by investigating the rate of the acquisition of computation skill and its relationship to arithmetic concept knowledge of the students. A positive relationship between rate of computation and concept score would strengthen the argument that concept learning should be emphasized in our schools.

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