THE CORRELATION OF SELECTED MATHEMATICAL MEASURES WITH PROBLEM SOLVING ABILITY

E. Ray Phillips and A. Edward Uprichard
University of South Florida

Herbert L. Johnson
Pinellas County Schools

SUMMARY

The ability to solve word problems in algebra by eighth and ninth graders was studied in relation to five selected variables (attitude toward math, algebraic skills, critical thinking, translation, and problem analysis). Algebraic skills and analysis of word problems correlated with ability to solve the problems. Additional findings are discussed.

INTRODUCTION

One of the major goals of mathematics instruction is to develop students' problem-solving ability. The Cambridge Conference on School Mathematics (Educational Services Incorporated, 1963) urged curriculum developers to place more emphasis upon the use of problems to introduce new mathematical ideas. However, as Kilpatrick (1969) implies, before these suggestions can be implemented much more research on problem solving is needed.

Problem solving has not been systematically investigated by mathematics educators. Few studies build upon previous research and even fewer are replicated. Gorman (1968) reviewed 293 studies on word problems conducted between 1925 and 1965. Only 37 of these studies were judged adequate in terms of internal validity. Suydam (1967) surveyed 84 studies of problem solving conducted between 1900 and 1965. Both Suydam and Gorman found conflicting results and indicated that many of these studies were plagued with design problems. Studies with adequate statistical designs yielded conflicting results or were of a highly theoretical nature with limited implications for classroom instruction. Loftus and Suppes (1972) and Suppes, Loftus and Jerman (1969) investigated factors related to problem difficulty. The results indicated that a problem is difficult to solve if (a) it is a different type from the problem that preceded it, (b) its solution requires a large number of operations, (c) its surface structure is complex and (d) it has a large number of words. Other studies deal with factors such as the effects of content of arithmetic word problems (Travers, 1967), the effect of language used in the problems (Steffe, 1967), and the effects of readability (Thompson, 1967).

In order to design instructional sequences which will facilitate the development of students' problem-solving ability, the relationship between an individual's success in problem solving and other instructional variables must be investigated. Obviously, success in solving word problems depends upon skills in reading and computation, however, the relative contribution of these skills has not been clearly explicated. Martin (1964) found reading comprehension, computation and abstract verbal reasoning were positively correlated with problem solving. Werdelin (1966), using factor analysis, found factors such as general reasoning and deductive reasoning related to problem-solving ability. However, verbal comprehension was unrelated to problem solving.

The present study was designed to investigate the relationship between ability to solve word problems in algebra by eighth and ninth graders and five selected factors. Specifically, the study, using a linear regression analysis, investigated the relationship between problem-solving ability and the following variables:

1. Attitudes toward mathematics
2. Algebraic skills
3. Critical thinking
4. Translation
5. Problem analysis

The research literature showing the relationship between problem solving and attitude toward mathematics is sketchy. Conflicting results are reported concerning correlation between problem solving and critical thinking and computational skill. Further, the effects of other factors such as the students' ability to analyze a problem and translate from English language to mathematical equations on problem solving has not been thoroughly investigated. It was the intent of the investigators to shed some light on these concerns.

METHODS

The Ss were 112 eighth graders and 123 ninth graders enrolled in public schools of Pinellas County Florida. Five different schools were involved with one intact class at each grade level participating in the study.

Four tests were constructed to assess ability to perform problem-solving tasks associated with the solution of word problems in algebra.

(1) Algebraic Skills: This test was designed to measure proficiency in algebraic skills needed to solve word problems in first year algebra. These included solving linear equations, quadratic equations, systems of linear equations, factoring, and simplifying algebraic expressions involving the four basic operations.

(2) Translation: A measure of ability to translate ordinary English and mathematical English into mathematical statements.

(3) Analysis: Students were asked to analyze given problem situations by answering a series of questions relating to each. The test was constructed to determine the ability to see relationships between variables, express these relationships in equations and recognize superfluous and pertinent data.

(4) Word Problems: A 50 minute test consisting of 5 word problems of varying difficulty ranging from very simple to a fairly complex mixture problem. All problems were modeled after exercises found in first-year algebra texts.

Two other tests were included in the battery: (a) Attitude Scale (Dutton, 1956) and (b) Critical Thinking Form G (American Council on Education, 1951).

The battery of six tests was administered to 235 first year algebra students (112 eighth graders and 123 ninth graders). The tests were administered in random order. Ss had completed approximately eight months of instruction in first year algebra at the time of administration of the tests. The tests were not timed, however, due to class schedules no test exceded 50 minutes.

Data were analyzed using a linear regression model (Hays, 1963). Regression coefficients, multiple correlation R, regression equations and partial correlation coefficients were obtained separately for eighth and ninth graders. The relative importance of each of the five independent variables upon the dependent variable (ability to solve word problems in algebra) was determined singly and in combination.

RESULTS

The means and standard deviations on each measure for both grades 8 and 9 are given in Table 1.

TABLE 1

Means and Standard Deviations on All Tests

Variable	Grade 8 $\bar{X}$	Grade 8 S	Grade 9 $\bar{X}$	Grade 9 S
1. Attitude	53.95	18.10	43.92	17.03
2. Algebraic Skills	11.86	8.54	7.87	4.19
3. Critical Thinking	27.54	6.60	23.50	6.79
4. Translation	16.46	2.77	13.74	4.47
5. Analysis	9.16	4.47	6.01	4.12
6. Problem Solving	13.28	4.90	9.49	6.08

The intercorrelation matrices for grades 8 and 9, are presented in Tables 2 and 3, respectively. Of the five independent variables used at the eighth grade level, only attitude and critical thinking made significant contributions to the multiple correlation and appeared in the final regression equation. At the ninth-grade level, three of the independent variables made significant contributions to the multiple correlation: algebraic skills, translation, and analysis.

TABLE 2

Correlation Matrix for Grade Eight (N=112)

Variable	1	2	3	4	5	6
1. Attitude	1.00					
2. Algebraic Skills	-.01	1.00				
3. Critical Thinking	.05	-.08	1.00			
4. Translation	.14	.08	.36	1.00		
5. Analysis	.18	.01	.41	.42	1.00	
6. Problem Solving	.27	.10	.37	.32	.35	1.00

The resulting regression equations were:

*Eighth: $Y = 2.24 + .25X_1 + .36X_3$

Ninth: $Y = .021 + .30X_2 + .20X_4 + .26X_5$

The multiple correlation (R) for grade eight was .45 accounting for only 20% of the variance. Multiple R for grade nine was .64 accounting for 40% of the variance.

The relative importance of each of the five variables in predicting the dependent variable (ability to solve word problems) was determined in all possible combinations. Space constraints prohibit listing the results of all of these analyses. The combinations of two, three and four variables producing the largest multiple R for each grade level are listed in Table 4.

TABLE 3

Correlation Matrix for Grade Nine (N=123)

Variable	1	2	3	4	5	6
1. Attitude	1.00					
2. Algebraic Skills	.12	1.00				
3. Critical Thinking	-.09	.33	1.00			
4. Translation	.02	.48	.47	1.00		
5. Analysis	.17	.62	.38	.61	1.00	
6. Problem Solving	.18	.56	.24	.50	.57	1.00

TABLE 4

Multiple Regression Combinations of Variables

	Combination of Variables	Multiple R
Grade 8	1,3	.45
	1,3,5	.49
	1,2,3,5	.50
Grade 9	2,5	.63
	2,4,5	.64
	2,3,4,5	.65

*Subscripts refer to the variables listed in Table 1.

DISCUSSIONS AND CONCLUSIONS

The means on all measures were higher for the eighth graders than for the ninth graders. One could speculate that this was because most students studying algebra in the eighth grade are generally the more capable students in mathematics. Attitude toward mathematics contributed significantly to the multiple correlation for the eighth grade but did not for the ninth grade. Again one might expect students studying algebra in the eighth grade to have a positive attitude towards mathematics. Attitude correlated the lowest of all variables under study with problem solving ability of ninth graders (r=. 18).

At the ninth grade level a very clear pattern emerges. Algebraic skills, translation from ordinary English to the language of mathematics, and analysis of word problems correlate consistently (r_. 5) with the student's ability to solve verbal problems. In general these mathematical factors seemed to be important correlates with problem solving ability. Certainly, instructional activities in the classroom should provide for maximal acquisition of these skills which might facilitate the development of the student's ability to solve word problems in algebra.

Two unexpected results appear. Critical thinking correlated low (r= .24) with ninth graders ability to solve word problems while algebraic skills correlated low (r=. 10) with eighth graders problem solving skills. It was noted that on many of the eighth grader's tests that Ss solved the problem without setting-up an equation. They analyzed the problem using drawings, logical guessing, and trial and error. These were recorded as correct responses whether an equation was employed or not. Thus, several students may have received a high score on the problem solving test and possibly a low score on the algebraic skills test. In general, one would not expect students to be able to solve complex verbal problems without being able to carry out simple algebraic manipulations. It seems clear that more research on mathematical correlates with problem solving is needed.

In interpreting these results, one must keep in mind that the tests on translation, analysis, and problem solving are measuring, in part, something in common. Also it is generally believed that reading comprehension is an important factor in facilitating problem solving ability. However, the relationship between reading comprehension and problem solving has not been clearly explicated. Thus, the inclusion of this variable might have strengthened the study.

REFERENCES

Dutton, W. H., Attitudes of junior high pupils toward arithmetic. School Review, 1956, 64, 18-22.

Educational Services Incorporated. Goals for School Mathematics. Report of the Cambridge Conference on School Mathematics, Boston: Houghton Mifflin, 1963.

Gorman, C. J., A critical analysis of research on written problems in elementary school mathematics. Dissertation Abstracts 28:4818A-19A, 1968.

Hays, W. L., Statistics for psychologists. New York: Holt, Rinehart and Winston, 1963.

Kilpatrick, Jr., Problem solving in mathematics. Review of Educational Research, 1969, 39, 523-534.

Loftus, E. F. & Suppes, P., Structural variables that determine problem-solving difficulty in computer-assisted instruction. Journal of Educational Psychology, 1972, 63, 531-542.

Martin, M. D., Reading comprehension, abstract verbal reasoning, and computation as factors in arithmetic problem solving. Dissertation Abstracts 24:4547-48, 1964.

Steffe, L. P., The effects of two variables on the problem-solving abilities of first-grade children. (Tech. Rep. No. 21) Madison: University of Wisconsin Research and Development Center for Cognitive Learning, 1967.

Suppes, P., Loftus, E. F. and Jerman, M., Problem-solving on a computer-based teletype. Educational Studies in Mathematics, 1969, 2, 1-15.

Suydam, M. N., The status of research on elementary school mathematics. Arithmetic Teacher, 1967, 14, 684-89.

Thompson, E. N., Readability and accessory remarks: Factors in problem-solving in arithmetic. Unpublished Doctoral thesis. Stanford: Stanford University, 1967.

Travers, K. J., A test of pupil preference for problem-solving situations in junior high school mathematics. Journal of Experimental Education, 1967, 35, 9-18.

Werdelin, I., A synthesis of two factor analyses of problem solving in mathematics. Didokometary, No. 8. Malmo, Sweden: Malmo School of Education, 1966.

Sample Test Items for Investigator
Constructed Measuring Instruments

I. Algebraic skills:

1. Simplify: $8x - 9 + x + 5 - 3x$.

2. Find the solution set of $5x - 4 \neq 12 - 3x$.

3. Perform the indicated operations:

$$-6(4x - 9)$$

4. Solve the following system of linear equations:

$$3x + 4y = 26$$
$$8x - y = 11$$

II. Translation:

Translate the following English sentences to a number sentence.

1. Twice some number is 3 less than 4.
2. The sum of two numbers is 65.
3. The product of a number and 4 times the number equals 40 divided by the number.

III. Analysis:

1. A mathematics club has 5 less members than 3 times the number it had two years ago. If there are 22 members now, how many members did the club have 2 years ago? Answers the following:

   a) Represent the number of members in the club two years ago.
   b) Represent the present number of members.
   c) State two quantities which are equal and form an equation.
   d) What does the variable in this equation represent?

2. A druggist wishes to make an 8% solution of a certain drug. How many ounces of a 20% solution must he add to 20 ounces of a 5% solution to get the desired 8% solution? Answer the following:

a) If x represents the number of ounces of the 20% solution, represent the number of ounces of the 8% solution.
b) How do you represent 8% of your answer to a) above?
c) Does your answers to b) represent the amount of the drug in the solution? Yes, No.
d) The druggist is adding 20 ounces of a 5% solution to x ounces of a 20% solution. Represent this mathematically.
e) What does your answer to d) represent in terms of the desired solution?
f) Do your answers to c) and d) represent the same quantity? Yes, No.
g) Based on the above analysis write an equation which will yield the value of x when solved.

IV. Word Problems:

Solve the following problems.

1. A missle traveling 9,000 miles per hour is shot at a fighter plane 100 miles away. If the airplane is going away from the missile at 300 miles per hour, how far will the plane fly before the missile will overtake it?

2. A candy store operator wishes to have a mixture of candy that will sell for 80¢ a pound. How many pounds of 90¢ candy must he mix with 1,000 pounds of 60¢ candy to obtain the desired mixture?

3. The difference of two numbers is 4-1/2 and one of the numbers is 5/8 of the other. Find the numbers.

4. How long will it take Bill and Frank together to build 200 ft. of fence if Bill can do the work alone in 4 days and Frank needs 5 days?