

## THE DEVELOPMENT OF OPTIMAL

### INSTRUCTIONAL SEQUENCES

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An optimal instructional sequence can be defined as one which maximally facilitates achievement, transfer, and retention; requires the least amount of time for the learner to complete; and induces minimal anxiety and frustration on the learner's part (Phillips, 1971). Two questions arise logically from the formulation of the above definition. (1) Do optimal instructional sequences exist? (2) How are they determined and verified? The purposes of this paper are to review the empirical evidence supporting the feasibility of the development of optimal instructional sequences and to explore the problem of developing and validating such sequences.

#### Learning Hierarchies

Gagné has characterized a learning hierarchy as "representing the most probable expectation of greatest positive transfer for an entire sample of learners for whom we know nothing more than what specifically relevant skills they start with" (Gagné, 1968). Of all the learning theories proposed today, the hierarchical theory seems to be the most potentially useful for practical classroom situations.

Numerous studies support the hierarchical structure of learning theory. DiVesta and Walls (1967) demonstrated positive transfer from relevant "pre-utilization" training to the Maier two-string problem. Davis (1967) showed the effectiveness of transfer of previously learned verbal rules to switch-light problems, and a similar theme is developed by Overing and Travers (1966, 1967) in studies pertaining to the problem of hitting an underwater target. Scandura and Wells (1967) demonstrated positive transfer effects from prior learning in concrete situations involving relevant rules in problems concerning mathematical groups and combinatorial topology. Battig (1968) cited evidence that the learning of paired associates is typically facilitated by prior discrimination learning on stimulus-terms and response-terms, as well as by prior learning on stimulus coding responses. For a more extensive review of studies related to transfer of training, Schultz (1960) may be consulted.

Another type of study from which evidence

about learning hierarchies may be derived is one which attempts to try out a total hierarchy in which the various levels of intellectual skills are to be learned in a single instructional sequence. Gagné (1963) hypothesized that "an individual will not be able to learn a particular topic if he has failed to achieve any of the subordinate topics that support it." This hypothesis was tested in several studies of the aforementioned type. Gagné and Paradise (1961) studied transfer relationships within a learning hierarchy for the task of solving linear equations. They concluded that when particular subordinate skills required for new learning were present in the learner high positive transfer resulted. When they were absent, very low transfer took place. In a similar study, Gagné, Mayor, Garstens, and Paradise (1962) measured the effects of learning program variations upon achievement. The results indicated that acquisition of subtasks at successively higher levels of the hierarchy was dependent upon prior mastery of subordinate subtasks. The authors also found that when the mediating effects were examined for subtasks intervening between higher and lower ones, proportions of achievement of higher subtasks indicated significant amounts of positive transfer. Several other studies (Gagné, 1962; Gagné, 1963; Gagné and Staff, University of Maryland Mathematics Project, 1965) yielded the same results reinforcing those of the previously discussed studies. In general, the results of these studies indicate that new skills and knowledge emerge from lower order knowledge, and that there is a significant amount of positive transfer from each successive subordinate level to the next higher level in a hierarchical ordering of such levels.

#### Sequencing Instructional Materials

Instructional design requires decisions about structuring content and designing and ordering a set of instructional tasks. Gagné (1967) and Briggs (1968) have proposed the use of instructional sequences that require the learner to follow a specific route through a content structure, suggesting that optimal instructional sequences can be developed by sequencing materials according to learning hierarchies.

Substantial evidence suggests that optimal learning sequences can, in fact, be determined. Recent studies of sequencing (Brown, 1970; Neidermeyer, Brown, and Sulzen, 1969; Phillips and Kane, 1973; Callahan and Robinson, 1973) indicated that Ss using materials sequenced according to learning hierarchies performed reliably better than Ss using materials whose sequence was scrambled, relative to time to complete the instructional program, to errors made on the program, and to performance on a criterion test of complex problem-solving skills. Brown (1970) concluded that when a sequence involves tasks that are complex, ordering of problem-solving behaviors is an important factor in learning, even for bright and relatively mature learners. In summarizing research on sequencing mathematical tasks, Miller (1969) concluded that mastery of individual subtasks in a hierarchy can be achieved in several ways, including learning from randomly ordered sequences, but that logical sequencing appeared best in terms of overall efficiency and effectiveness. Roe (1962, p. 409) stated that "careful sequencing of items has a significant effect on student performance, at least for programs of some length and complexity." King (1970) described the above as key studies utilizing programs based on hierarchies and well controlled learning situations avoiding methodological weaknesses.

Several studies (Levin and Baker, 1963; Miller, 1965; Payne, Krathwohl, and Gordon, 1967; Roe, Case, and Roc, 1962) however, suggest that varying sequences of instructional stimuli that have high interdependency does not make much difference in effectiveness of instruction. However, some of these studies are plagued with design problems (King, 1970). Before sequencing instructional materials for use in classrooms, the effects of sequence upon time to achieve the terminal behavior, achievement, transfer, and retention should be investigated.

#### Hierarchy Validation Techniques

Validating a learning hierarchy is not a simple undertaking. Many researchers (Ausubel, 1963; Bruner, 1964; Gagné, 1965; Glaser, 1964; Suppes, 1966) have long recognized that sequence is a critical variable in learning. The learner begins with simple tasks and progresses to increasingly complex tasks. However, both Gagné (1968) and Pyatte (1969) have pointed out that determination of this hierarchical ordering of subtasks from simplest to most complex is a major problem.

Gagné and Paradise (1961) were pioneers in learning hierarchy validation. Their approach was direct validation based on learners' responses to a programmed learning sequence and criterion tests administered immediately after the instructional program to establish pass-fail patterns for each component of the learning hierarchy. Consider the simple two-level hierarchy in Figure 1.

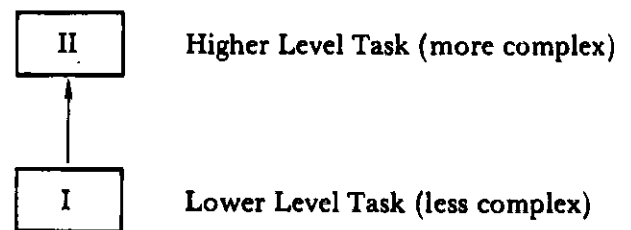


Figure 1. A Simple Two-Level Hierarchy.

Gagné's validation procedure was based on the assumption that task I must be mastered before task II can be mastered. Failure on task I would automatically produce failure on task II. Using + and - to represent pass and fail respectively, there are four possible pass-fail relationships which can be observed: (++) , (+-) , (--), (-+). For example, the first relationship signifies that the learner passed (performed to criterion) both task I and task II. Only the relationship (+-) is in direct contradiction of the theory and indicates a flaw in ordering. The relationship (-+) (passed lower level task but failed higher level task) indicated a weakness in the instructional program but provides no information concerning the validity of the hierarchy.

To validate a hierarchy, Gagné analyzed the pattern of responses of each transfer in the hierarchy. That is, he constructed a contingency table of the observed responses to a higher level task and the task immediately prerequisite to it as illustrated in Figure 1. He calculated the following ratio to determine the degree of validity of the hierarchy.

$$\text{Proportion of Positive Transfer (P+)} = \frac{++ + --}{++ + -- + +-}$$

Perfect validity would be indicated by a ratio of 1.00. If all learners contradicted the theory, having observed patterns (+-), then the ratio would be zero. Thus, P+ is bounded above and below by 1 and 0 respectively. The degree of validity of any hierarchy is measured by P+ with the lower limit of acceptability for P+ being .90.

Phillips and Kane (1974) investigated the efficacy of this ratio when applied to test data alone. Using Gagné's task analysis, a learning hierarchy for the computational skills of whole number addition was constructed. Based upon the hypothesized ordering of the subordinate levels, a test was constructed to assess mastery at each level. A second test utilizing a random ordering of the same items was constructed. Both tests were administered to a large sample of elementary school children in grades 3 through 6 in order to obtain a wide range of ability levels. The proportion of positive transfer between adjacent items on both tests was above .90, except in two instances. Thus both the hypothesized and the random hierarchies were validated by this procedure. The authors concluded that prior educational experiences confounded the issue of positive transfer when considering test data alone.

Walbesser (1968) has refined and extended Gagné's approach to hierarchy validation. Task analysis is used to generate hierarchies of subordinate subtasks. Learning sequences are designed to correspond to the hypothesized hierarchies. Pass-fail contingencies are used to test dependency of each individual task on its immediate prerequisite subtasks. The staff for AAAS pointed out that a high proportion of positive transfer—Gagné's statistics—is a necessary but not a sufficient condition for a valid hierarchy. Using the pass-fail relationships defined by Gagné, the AAAS defined the following three ratios:

$$(1) \text{ Consistency ratio} = \frac{++}{(++) + (+-)}$$

$$(2) \text{ Adequacy ratio} = \frac{++}{(++) + (-+)}$$

$$(3) \text{ Completeness ratio} = \frac{++}{(++) + (--)}$$

Ratio (1) is a measure of how consistent the data are with the hypothesized dependency. Ratio (2) is a measure of the adequacy of the identified subordinate tasks. Ratio (3) is a measure of the effectiveness of instructional materials designed to bring about learning. In the development of *Science—A Process Approach*, the AAAS (1968) has considered high consistency, adequacy, and completeness ratios as the necessary and sufficient set of characteristics for a valid hierarchy. No significance test has been developed for either Gagné's ratio or those defined by the AAAS. Eisenberg and Walbesser (1971) have further

refined this validation technique which includes the use of six different ratios.

Cox and Graham (1966) used the Guttman Scalogram Analysis (1944) to develop a sequentially scaled achievement test. Essentially, the Guttman technique (Torgerson, 1958) orders items such that a learner's response pattern to the set of items can be predicted from knowledge of a learner's total score. The coefficient of reproducibility which is given by one minus the ratio of the total number of errors to total number of responses.

$$\text{Rep} = 1 - \frac{\text{total number of errors}}{\text{total number of responses}}$$

indicates the degree to which a set of items forms a perfect scale. Error is defined as instances where a subject passes a higher level item after failing a lower level prerequisite item. Guttman suggested .90 as an acceptable lower limit for Rep. Cox and Graham reported a reproducibility coefficient of .97 for their final arrangement of items and thus concluded their hierarchical ordering as valid.

Several investigators have attempted to use item difficulty as a means of validating a hierarchy. Studies by Stoker and Kropp (1964) and Herron (1965) showed that the cognitive skills in the *Taxonomy* were more difficult than those at lower levels in the *Taxonomy*. Kropp, Stoker, and Bashaw (1966) concluded that their results based on item difficulty validated the *Taxonomy*. Miller and Phillips (1975) used item difficulty in conjunction with the Walbesser technique to validate a hierarchy for the computational skills of rational number subtraction.

Grib and Rimoldi (1960) developed a procedure for comparing two patterns of responses from several subjects using the index of agreement. Listing each subject's responses to a set of items produces an observed matrix of responses with rows corresponding to items. An expected matrix can be formed based on an operational definition of what response patterns are expected from a given set of items. The only restriction on the expected matrix is that the subject's total score on the expected pattern must equal his total score on the observed pattern. These two patterns can be compared and an index showing the amount of agreement between the two patterns can be computed. Several studies employing this technique are reported in the literature (Harke, 1970; Miller and Phillips, 1975; Uprichard and Phillips, 1976; Uprichard and Phillips, 1977).

There are several methods of hierarchical analysis reported in the literature which are used in the generation of hierarchies rather than the validation of deductively analyzed hierarchies (Resnick and Wang, 1969). In these methods the data must speak for itself with no *a priori* assumptions concerning order.

### Conclusions and Recommendations

In reviewing the literature, one finds substantial evidence to support the general theory of the hierarchical structure of knowledge (studies cited previously). There seems to be little doubt that new skills and knowledge emerge from lower order knowledge, and that there is a significant amount of positive transfer from each successive subordinate level to the next higher level in a hierarchical ordering of such levels. The studies on sequencing instructional materials reviewed suggest that the sequence of subordinate tasks in a learning hierarchy, after sufficient validation, can describe a teaching program that will effectively accomplish the instructional objectives. That is, an instructional sequence based on the levels in the hierarchy will represent an optimal route for acquisition of the terminal task by a sample of learners. Hence, it appears that the development of optimal instructional sequences becomes essentially a problem of validating a learning hierarchy.

The question of validating the ordering of the levels in a hierarchy is a complex and elusive problem. Gagné (1968) stated that various methods have been tried but none seems entirely satisfactory as yet. Several of these techniques were reviewed in this paper; and undoubtedly, others are being developed and tested. It seems imperative that efforts to develop more efficient procedures for validating learning hierarchies be continued.

Several questions have been raised regarding Gagné's method of empirical validation of the hypothesized ordering of a set of subordinate tasks (Walbesser, 1968; Phillips and Kane, 1974; Phillips, 1971; White, 1973). These include: small sample size, imprecisely defined use of task analysis, no significance test for the proportion of positive transfer ratio, and errors of measurement.

White (1973) has suggested two lines of further learning hierarchy research.

- (1) Use Gagné's model and try to minimize the above problems.
- (2) Use White's (1973) more rigorous model.

White's method essentially states, for a set of  $k$  subtasks, teach them in all  $k!$  possible orderings to different groups. Based on the performance of the learners in each group, one determines the "best" ordering. This ordering represents a valid hierarchy. Obvious difficulties with this method, of course, are the large number of experimental groups that would be involved, the number of subjects needed, and the time involved in this type of validation. When the number of subtasks is large, White suggested that the researcher must resort back to Gagné's original design with some recommended changes. One of the more important of the recommendations is to replace indices such as Gagné's "proportion of positive transfer," and the ratios proposed by Walbesser and Guttman's index of reproducibility, with a statistical test of hierarchical dependency. White and Clark (1973) report the development of such a test.

Validating a learning hierarchy by either of White's suggested procedures is a tedious and costly undertaking. In either case, one must teach a sequence of tasks to a large sample of learners; and with the rigorous model,  $k!$  sequences must be presented to different groups of learners. Based upon the performance of the learners in the experimental groups, the best ordering is determined as the valid sequence. Thus, it seems that White would define a valid learning hierarchy similar to Kane, McDaniel and Phillips (1971). That is, a valid hierarchy is one which yields an optimal learning sequence.

Phillips (1971) labeled validation techniques that are based on test data alone as indirect procedures. It seems that the development of less expensive indirect validation techniques could greatly improve the use of learning hierarchies in designing instructional sequences.

A hypothetical case is considered below to illustrate how one might use an indirect procedure to validate a learning hierarchy. Suppose 20 subtasks are to be ordered hierarchically, and for simplicity's sake, item difficulty (Nunnally, 1967) will be used as the basis of ordering. (The author is not assuming nor proposing that item difficulty is an adequate validation procedure. This is for purely illustrative purposes.)

1. Construct a test with 3 items for each of the 20 subtasks. (A passing grade for each subtask will be correct responses to 2 of the 3 items for that subtask.)
2. Administer the test to a large sample of subjects to include a wide range of ability

and achievement levels.

3. From the test results, calculate the item difficulty for each of the 20 items and order from highest to lowest. This will represent the "valid" ordering of the 20 subtasks.

Regardless of the validation technique used with the three steps above, one must have some evidence of the adequacy of the techniques. That is, did the technique employed yield a valid hierarchy? Are the subtasks now in an order which would lead to an optimal instructional sequence? Such evidence may be obtained in the following manner:

1. Design instructional materials for each of the 20 subtasks.
2. Sequence these materials according to the hierarchy generated by the indirect procedure under scrutiny (in this case item difficulty).
3. Sequence the instructional material for each of the 20 subtasks in several other orderings generated by direct or indirect methods, as well as sequences ordered from logical analysis of the content and those deliberately ordered in a manner which one believes would interfere with learning.
4. Randomly assign a large number of students to each of these groups and teach each group using one of the sequences of instructional materials.
5. Upon completion of the instructional program, compare each group on achievement, retention, transfer, attitude toward the program, and time needed to complete the program.
6. If the indirect procedure produces an optimal learning sequence, one concludes the

procedure is adequate for generating a valid hierarchy. (Keep in mind, of course, that more than one valid hierarchy could be generated from the same 20 subtasks.)

Now that the procedure has been put to the ultimate test of actually sequencing instructional materials based on the hierarchy generated, one can apply the indirect procedure to a different set of subtasks with some confidence. That is, there's no need to rediscover the wheel each time a hierarchy is to be validated. The indirect procedure can be applied to test data alone without going through a costly instructional program to validate directly.

Phillips and Kane (1973) used the above technique with instructional sequences generated by seven different hierarchy validation techniques. The effects of sequence upon the overall efficiency of learning from programmed mathematical materials was investigated. While no sequence maximally facilitated achievement, retention, and transfer, and at the same time required less time to complete, Ss using the materials sequenced via the hierarchy validated by the Walbesser Technique performed significantly better than Ss in any of the other six sequences.

Although there is considerable room for improvement in the Walbesser Technique, the results of studies indicate that it is an acceptable indirect validation technique (Phillips, 1971; Miller and Phillips, 1975; Uprichard and Phillips, 1975). Hopefully new hierarchy validation techniques will be developed which are more readily adaptable to the indirect approach to validating a learning hierarchy. With the development of adequate hierarchy validation techniques, the possibility of the development of optimal instructional sequences becomes a very real one.

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