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Elaboration of HLM Growth Modeling Results

Richard L. Tate

Florida State University

Standard reporting of the modeling of individual growth or change curves with hierarchical linear models (HLM) typically includes a focus on certain important results (e.g., the variance of the status of the outcome) at a single time in the growth curve, a time that is determined by the specification of the origin of the time scale. It is argued here that such reporting should be extended to show the variation of these important results over the time span of the study. The required procedure, involving only some simple matrix algebra and a technical graphics program, is illustrated with data for the nonlinear growth of reading ability for young children.

The growth curve modeling application of the hierarchical linear models (HLM)[1] technique has received serious attention over the last decade.[2] Authors have articulated convincingly the potential advantages of HLM over more traditional methods, emphasizing the associated conceptual elegance and ability to address long standing problems in the assessment of change (e.g., Bryk & Raudenbush, 1987; Rogosa, Brandt, & Zimowski, 1982; Rogosa & Willett, 1985). As a result, the modeling of growth with HLM is being used increasingly in substantive research (e.g., Foorman, Francis, Novy, & Liberman, 1991; and Huttenlocher, Haight, Bryk, & Seltzer, 1991).

For the analyst wishing to learn more about HLM modeling of growth, seminal articles (e.g., Bryk, et al., 1987; Goldstein, 1986a, 1986b;

Longford, 1987: and Raudenbush, 1988) and several accessible books are available (e.g., Byrk & Raudenbush, 1992; and Goldstein, 1987). Moreover, "methods diffusion articles" encouraging the use of the new technique in different disciplines are appearing (e.g., Francis, Fletcher, Stuebing, Davidson, & Thompson, 1991; and Tate & Hokanson, 1993). For methodologists, healthy critical debate of the strengths and limitations of the approach is ongoing (see, e.g., the summer 1995 issue of the *Journal of Educational and Behavioral Statistics*), with attention to remaining tasks important to the appropriate use of HLM for growth modeling (e.g., Rogosa & Saner, 1995). Finally, perhaps most importantly for the applied analyst, user-friendly dedicated computer programs are available (e.g., Bryk, Raudenbush, Congdon, & Seltzer, 1988; Longford, 1990; and Prosser, Rasbash, & Goldstein, 1991), and a comparative review of some of the programs has been offered (Kreft, de Leeuw, & van der Leeden, 1994).

The content of the reporting of HLM growth modeling results appears to be relatively well established. Standard reporting would start with results from analyses required to properly specify the within-subject growth model. For example, if a polynomial model were being used to fit nonlinear growth curves, the tests determining the appropriate order of the polynomial would be reported. Then it would be customary to consider an "unconditional" between-subject model providing the basis for estimation of the true variance over subjects of the parameters defining the within-subject model. If there is evidence that one or more of the parameters vary over subjects, a "conditional" model would be used to explain that variation with one or more subject characteristics.

In standard reporting, special attention is usually given to individual model parameters reflecting results at a time equal to zero in the growth curve. For example, results for individual parameters in the unconditional model provide the variance of the outcome at time equal to zero, a quantity of traditional interest in the assessment of change literature. By specifying the centering of the time scale, the analyst can determine the time associated with this important result. For example, direct determination of the variance of the *initial* status of the outcome is provided by setting time equal to zero at the initial time of measurement. Different results would be obtained for the *final* outcome status if time were set equal to zero at the last measurement. Whatever choice is made for the centering of time, it appears to be customary to present these results at only one point in time.

It is suggested here that the reporting of critical growth curve results at only one point in time is unnecessarily restrictive. Instead, the description of these results should be routinely extended to show their variation continuously over time. As illustrated below, this elaboration provides useful information at minimal cost in effort. Specifically, the illustration here considers a nonlinear growth curve and includes graphical representations of the following results:

- variance of the outcome status over time,
- variance of the growth rate of the outcome over time,
- correlation of the outcome status and the outcome growth rate over time,
- stability of the outcome over time (i.e., the correlation of outcome status at one time with that for a later time),

- effect of an individual attribute on the outcome status over time, and
- effect of an individual attribute on the growth rate of status over time.

Before description of the procedure for the elaboration, the data used for the illustration will be briefly described.

Example. The proposed elaborations will be illustrated with data provided by Rick Wagner and colleagues. Briefly, the data are from a larger longitudinal study of the relationships over time between phonological processing skills of young children and their word-level reading ability (e.g., Wagner, Torgesen, Laughon, Simmons, & Rashotte, 1993). For this illustration, the outcome of interest is *Reading* as measured by standardized word-decoding tasks. Reading was measured using parallel forms each year from kindergarten to grade 3 for a sample of 223 children. There were no missing data. Potential explanatory variables measured at kindergarten were: *Verbal ability* -- vocabulary as measured by a standardized test; *Awareness* -- phonological awareness measured with tasks assessing the ability to analyze words into the constituent basic sounds or phonemes and the ability to synthesize phonemes into whole words; *Memory* -- use of phonological codes to represent information for short-term memory storage; *Isolated naming* -- retrieving phonological codes for items presented in isolation; and *Serial naming* -- retrieving phonological codes for test items presented serially. The correlations among the explanatory variables were positive and moderate in magnitude, ranging up to 0.46. More information on the variables and data collection is provided in Wagner, et al. (1993).

The software used for the illustration was ML3, Version 2 (Prosser, et al., 1991), and PSI-PLOT (Poly Software International, 1995) was the technical graphics software used to create the graphical representations.

Results

The specification of the individual growth curve is the starting point for HLM modeling of growth curves. For the example, inspection of a large number of individual student growth trajectories suggested that a typical trajectory was a curve with two inflection points, one in the earliest grades when a child began to read and one near grade 3 where there was a tendency for decreasing growth rate. It was therefore decided to use a third-order polynomial for the individual growth model to provide an adequate fit to the data (later tests supported this choice). That is, the true value of the status of Reading for the i^{th} subject, using the notation from Bryk and Raudenbush (1992), was modeled with

$$Status_i = \pi_{0i} + \pi_{1i}a + \pi_{2i}a^2 + \pi_3 a^3 \tag{1}$$

where a represented time and the π's were the coefficients of the polynomial. (A later section considers the approach described here for a simple linear growth model.) In order to allow an estimate of the error variance of the individual growth model, only the first three of the four coefficients in (1) were allowed to vary over individuals (indicated by the i subscripts for the coefficients). The coefficient for the third-order term (with no i subscript) was assumed to be constant over students.

An important part of the standard reporting of HLM results focuses on the constant, π_{0i}, in (1). As noted earlier, this coefficient is the status of the outcome when $a = 0$. Thus, it is customary to decide what time during the study would be of the most interest for the description of status, and to then center accordingly the time variable by setting a equal to zero at that time. For the current example, the time variable was centered on the time at which Reading was measured for all students in grade 3. Thus, time took on only the four values of -3 (kindergarten measure), -2 (grade 1), -1 (grade 2), and 0 (grade 3). With this scaling of a, the constant term in (1) was the final status of Reading at grade 3.

The true rate of Reading growth for the ith individual was expressed by the first derivative of (1) with respect to a, i.e.,

$$Rate_i = \pi_{1i} + 2\pi_{2i}a + 3\pi_3 a^2 \tag{2}$$

With the scaling of a, the constant term in (2), π_{1i}, was the rate of Reading growth for individual i at the time of the last testing in grade 3. (The rate of acceleration of Reading growth could also be obtained with the second order derivative of (1) with respect to time, but acceleration results are not included in the following illustration.)

The growth curve model in HLM allows variation from individual to individual; this is represented in (1) by the i subscript for the p model coefficients. As noted above, modeling of this variation was accomplished in two separate steps. First, an unconditional model was used to determine the estimated true variances of model parameters. Then, the true variation was modeled with individual characteristics in a conditional model. Each of these steps is considered separately below, describing first selected standard reported results and then presenting the proposed elaborations.

Unconditional model results

The unconditional model simply stated that the random π coefficients in (1) resulted from random variation about population means. For the current example, the corresponding models were $\pi_{0i} = b_{00} + r_{0i}$, $\pi_{1i} = b_{10} + r_{1i}$, and $\pi_{2i} = b_{20} + r_{2i}$. The b's are the population means and the r's are the random individual effects. (Remember that it has been assumed that the coefficient for the third order term is fixed, so $\pi_3 = b_{30}$.). The variances of the random effects r_{01}, r_{1i}, etc. are denoted t_{00}, t_{11}, etc., while the covariance between the j^th^ and k^th^ effects is denoted t_{jk}. Substituting these individual-level models into (1) and (2), the within-subject models for Reading status and the rate of growth can be expressed as

$$\begin{aligned} Status_i &= (\beta_{00} + \beta_{10}a + \beta_{20}a^2 + \beta_{30}a^3) + (r_{0i} + r_{1i}a + r_{2i}a^2) \\ Rate_i &= (\beta_{10} + 2\beta_{20}a + 3\beta_{30}a^2) + (r_{1i} + 2r_{2i}a) \end{aligned} \tag{3}$$

The fixed parts of the two equations in (3) (i.e., the parts involving the b parameters) represent the average growth trend and average rate of growth, respectively, for Reading in the population, while the random portions (in the second set of parentheses in each equation) represent individual variations about the average results.

Standard unconditional model results reported would include the estimates and standard errors of the fixed coefficients. For the current example, all fixed coefficients were statistically significant. For a nonlinear growth model, the reporting would probably include graphical representations of the fixed portions of the equations in (3), i.e., graphs of the average status trend and the average growth rate over the four years of the study (graphs not shown here).

The estimates and standard errors of the random effect variances would also be reported. Statistical significance for a random effect term is evidence of true variability in the population, true variability that is available for further modeling in a conditional model as described below. For the current example, the variances of the random effects for the zero, first, and second order terms were significant, indicating support for modeling of the terms with individual characteristics.

In addition, selected portions of the random effect results are typically of interest in their own right. It is customary to focus on the results for $a = 0$ provided by the random terms r_{0i} and r_{1i} in the first and second equations, respectively, in (3). Specifically, the variance of r_{0i} is the variance of the status of the outcome at $a = 0$, the variance of r_{1i} is the variance of the rate of growth of the outcome at $a = 0$, and the correlation of r_{0i} and r_{1i} (obtained with the corresponding covariance of the two terms) is the correlation of status and rate at $a = 0$. For the current example, the variance of the status of Reading at the grade 3 testing was 546.3 (standard deviation of 23.4), the variance of the rate of Reading growth at the same time was 91.56 (standard deviation of 9.57), and the correlation between the status and growth rate of Reading was 0.35.

An elaboration of standard random effect results.

The proposed elaboration of standard HLM reporting would extend the random effects results given above for a = 0 to describe graphically how they vary over the time span considered in the study. Such a description is made convenient with contemporary technical graphics software and some simple matrix algebra. Specifically, a matrix operation is used to express a quantity of interest (e.g., the variance of status) as a polynomial in a with the polynomial coefficients defined in terms of the estimated random effect variances and covariances. Entry of the resulting expression in any technical graphics program results in a graphical representation of the variation of the quantity with a.

To illustrate, consider the variance of Reading status as a function of time. It is seen from the first equation in (3) that the variance of the status of Reading at a specific value of a will be the variance of the expression in parentheses, i.e.,

$$Var(Status) = Var(r_{0i} + r_{1i}a + r_{2i}a^2) \tag{4}$$

The variance of a linear combination of random variables is computed with the matrix product **b'Vb**, where **b** is the vector of defining coefficients in the linear combination and **V** is the covariance matrix of the random variables. Using **b'** = (1 a a^2) from (4), the resulting matrix product is

$$Var(Status) = \tau_{00} + (2\tau_{10})a + (2\tau_{20} + \tau_{11})a^2 + (2\tau_{12})a^3 + (\tau_{22})a^4 \tag{5}$$

where the symbols for the random effect variances and covariances were defined earlier.

In the current example, the elements of the covariance matrix for the random effects, **V**, were: t_{00} = 546.3, t_{10} = 77.90, t_{11} = 91.56, t_{20} = -30.80, t_{21} = 18.39, and t_{22} = 7.837. Using PSI-PLOT (Poly Software International, 1995) or any other technical graphics package, the resulting standard deviation of the status of Reading can then be plotted as a function of *a* as shown in Figure 1. (The plot was not extended to the initial testing time [a = -3] because of inadequacies of the individual growth model near the origin of the Reading scale.) The increasing variability of Reading status over three years in Figure 1 is consistent with the "fan spread" hypothesis that individual differences increase over time.

In similar fashion, from the second equation in (3) the defining coefficients of the variance of the individual rate of growth are $\mathbf{b}' = (0\ 1\ 2a)$. The square root of the expression resulting from expansion of **b'Vb** is also plotted in Figure 1. The variability of the individual rate of growth is shown to vary somewhat over the span of the study.

The determination of the correlation between Reading status at time *a* and the rate of growth at another time a^* requires first the computation of the associated covariance with COV (Status, Rate*) = **b'Vc** where, from above, $\mathbf{b}' = (\ 1\ a\ a^2\ )$ and $\mathbf{c}' = (\ 0\ 1\ 2a^*\ )$. The expression for the correlation is then obtained by dividing the covariance expression by expressions for the standard deviation of status at time *a* (the square root of **b'Vb**) and the standard deviation of the rate at time a^* (the square root of **c'Vc**). Assume, for illustration, an interest in the correlation between the status of Reading at a given time and the simultaneous rate of growth ($a = a^*$). The resulting relationship plotted in Figure 2 indicates that the correlation is near perfect through the first year but then begins to decrease rapidly over the following two years.

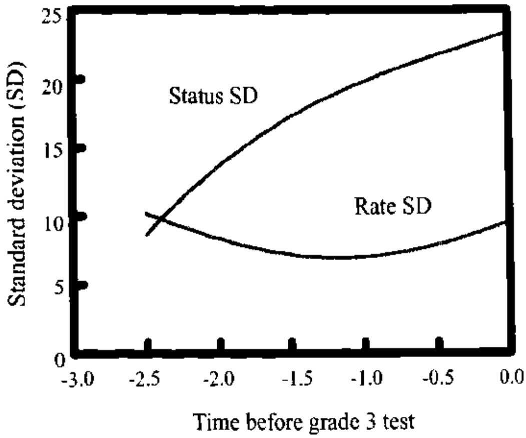

Figure 1. *Variation over time of the Standard Deviation (SD) of the status of Reading and the rate of growth of Reading.*

Stability of Reading status. When random variation of the coefficients in individual polynomial growth curves allows the "crossing" of individual growth trajectories (i.e., when the *relative* status of individuals may change over time), the researcher may be interested in the stability of individual outcome status. This result is obtained in a fashion similar to that shown above. The stability in individual status of Reading between time a and time a^*, represented by the correlation between status at the two times, is determined by dividing the expression for the covariance, COV(Status, Status*) = **c'Vc***, by the appropriate standard deviations (square roots of **c'Vc** and **c*'Vc***), where $\mathbf{c}' = (\ 1\ a\ a^2\ )$ and $\mathbf{c}^{*\prime} = (\ 1\ a^*\ a^{*2}\ )$. For the current example, when the stability of Reading status over a year (i.e., $a^* = a + 1$) was computed, the stability was virtually equal to one from $a = -2.5$ to $a = -1.0$ (graph not shown). This near perfect stability of relative status was consistent with modeled individual growth curves that demonstrated very little crossing of individual trajectories.

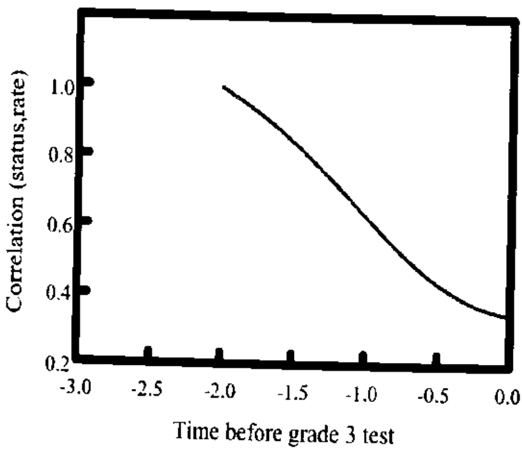

Figure 2. *Variation over time of the correlation of the status of Reading and the simultaneous growth rate of Reading.*

Conditional model results

The modeling of the estimated true variability of the random coefficients in (1) is accomplished with a conditional model that expresses each coefficient as a function of individual characteristics, i.e., for the example,

$$
\begin{aligned}
\pi_{0i} &= \beta_{00} + \beta_{01}X_{1i} + \ldots + \beta_{05}X_{5i} + r_{0i} \\
\pi_{1i} &= \beta_{10} + \beta_{11}X_{1i} + \ldots + \beta_{15}X_{5i} + r_{1i} \\
\pi_{2i} &= \beta_{20} + \beta_{21}X_{1i} + \ldots + \beta_{25}X_{5i} + r_{2i} \\
\pi_{3i} &= \beta_{30}
\end{aligned}
\tag{6}
$$

where the Xs are the individual characteristics measured at kindergarten that were defined earlier, i.e., X_1 = Verbal ability, X_2 = Awareness, X_3 = Memory, X_4 = Isolated naming, and X_5 = Serial naming. The Xs were standardized so that the intercepts in (6) represented the growth polynomial coefficients for an average individual.

The estimates of the fixed coefficients in (6) for the example are given in Table 1. The coefficients of the first equation in (6) are in the first column of the table, those for the second equation are in the second column, and so on. (Terms not shown in the table were not significant in initial analyses and were dropped from the model in a backward elimination process.) The centering of the time variable at grade 3 allows the direct interpretation of some of the results in Table 1. For example, the effects in the "constant term" column reflect the unique effects of the Xs on the status of Reading at grade 3 (see [1] for a = 0). It is seen that all effects were positive in direction, as expected, with all but the effect for Serial naming being statistically significant. The results in the "linear term" column of Table 1 give the effects of the Xs on the growth rate of Reading at grade 3 (see [2] with $a = 0$). The effects for Verbal ability, Memory, and Isolated naming on the growth of Reading at grade 3 were positive and statistically significant. The effects of Awareness and Serial naming on the growth rate were negative and statistically significant.

Table 1
Estimated Effects for the Conditional Model

Variable	Constant	Linear	Quadratic
Verbal ability (X_1)	4.18*	1.47*	-
Awareness (X_2)	4.75*	-2.83*	-1.23*
Memory (X_3)	3.79*	1.27*	-
Isolated naming (X_4)	5.41*	2.04*	-
Serial naming (X_5)	3.01	-2.57*	-0.96*
Constant	58.3*	10.0*	-13.4*

* z statistic > 2

Another approach to the description of the effects of the Xs on the individual growth curves would use graphical representations to indicate the impact of each X on the entire individual Reading trajectory. For example, Figure 3 shows two estimated growth curves reflecting the effect of a plus and minus one standard deviation change on Awareness, holding all other variables constant at the means (this figure was obtained by substitution of the results from Table 1 into the fixed portion of the first equation of [3]). Since the Reading scale is considered to be approximately a ratio scale with the zero point reflecting virtual absence of reading ability, Figure 3 suggests that Awareness at kindergarten has an important effect on the "start time" when the child begins to read, with increasing Awareness associated with earlier start times. The two trajectories are approximately parallel over much of the time span of interest, indicating that Awareness differences at kindergarten are not contributing to the increasing variability of Reading over time. A similar pattern was found for the Serial naming variable measured at kindergarten.

A different type of effect was obtained for the Isolated naming variable as shown in Figure 4. This variable did not have any perceptible effect on the time when the child begins to read. However, because of the positive effect of Isolated naming on the *growth rate* of Reading, it did have an appreciable effect on the variability of Reading over time. Similar patterns were found for the Verbal ability and Memory variables.

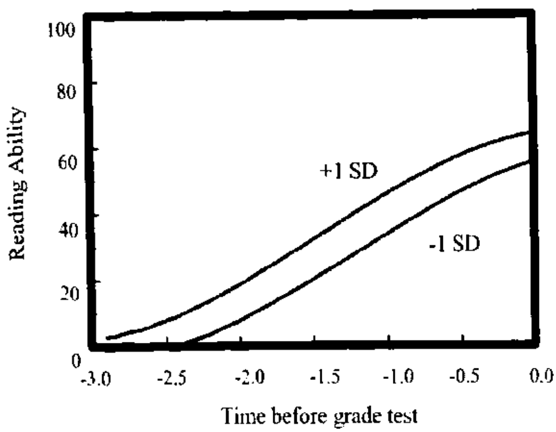

Figure 3. *The effect of Awareness on Reading growth as represented by two predicted trajectories, one for children one standard deviation above the mean on Awareness at kindergarten and one for children one standard deviation below the mean.*

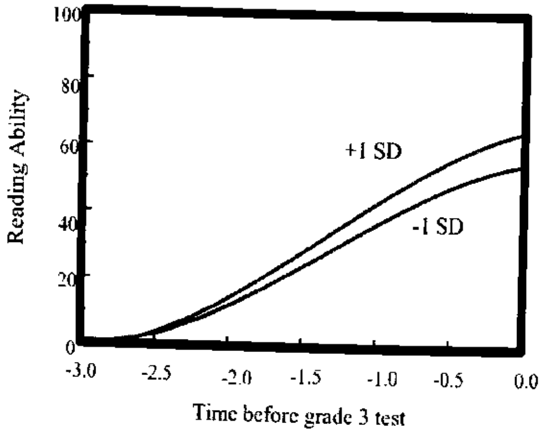

Figure 4. *The effect of Isolated naming on Reading growth as represented by two predicted trajectories, one for children one standard deviation above the mean on Isolated naming at kindergarten and one for children one standard deviation below the mean.*

Elaboration of the conditional model results.

The standard results for $a = 0$ given above could be extended by describing the effects of the Xs on the status and growth rate of Reading over the entire range of a. For the jth X, the effect on the status of Reading would be obtained by substituting (6) into (1) and taking the first order partial derivative with respect to X_j. The result is

$$Effect\ of\ X_j\ on\ Status = \beta_{0j} + \beta_{1j} + \beta_{2j}a^2 \tag{7}$$

The curves for all of the Xs are plotted in Figure 5. For Awareness (X_2) and Serial naming (X_5), the effects were relatively strong and positive over the entire time span. The large positive effect at the earliest times implies the effect on start times shown in Figure 3. The decreasing size of the effect at the end of the study is consistent with the pattern of slight convergence shown in Figure 3. For the other three Xs (Verbal ability [X_1], Memory [X_3], and Isolated naming [X_4]) the effects were virtually zero at the earliest ages and then increased to larger positive values at the end of the study. The initial zero effects are consistent with the lack of an effect on the start time (see Figure 5), and the steadily increasing effects over time imply contributions to the increasing variability of Reading over time. (Similar procedures could be used to superimpose confidence bands on the curves of Figure 5 if the necessary covariances for the fixed coefficients are provided by the HLM software.)

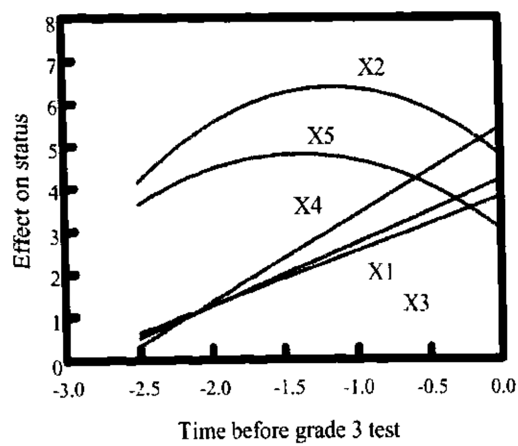

Figure 5. *The effect of five attributes at kindergarten on the status of Reading over time (X1 = Verbal ability, X2 = Awareness, X3 = Memory, X4 = Isolated naming, and X5 = Serial naming.*

Finally, similar descriptions can be provided for the rate of growth. Substitution of (6) into (2) and partial differentiation with to X_j results in

$$\text{Effect of } X_j \text{ on Growth Rate} = \beta_{1j} + 2a\beta_{2j} \tag{8}$$

The resulting plots are shown in Figure 6. The results are consistent with the picture provided by Figure 5. For example, the negative effects of X_2 and X_5 on growth rate towards the end of the study imply the convergence pattern seen in Figure 4. Also the sizes of the constant positive effects of X_1, X_3, and X_4 reflect the relative contributions of these variables to the increasing variability of Reading over time.

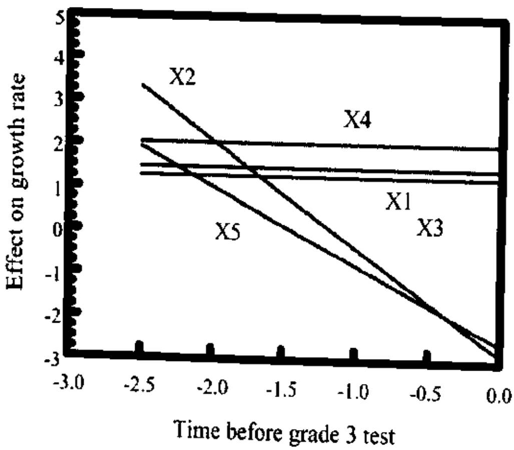

Figure 6. *The effect of five attributes at kindergarten on the growth rate of Reading over time (X1 = Verbal ability, X2 = Awareness, X3 = Memory, X4 = Isolated naming, and X5 = Serial naming.*

Procedure for linear growth models

The proposed procedure for elaboration illustrated above becomes simpler when individual data can be fit adequately with a linear growth model. The corresponding revisions in the procedure start with the use of a linear model for status in (1), dropping the last two terms in the nonlinear equation. The rate of growth for the ith individual in (2) is then a constant (p_{1i}). In the elaboration of standard random effect results, the variance of the status in (4) simplifies to the second order equation of t_{00} +($2t_{10}$)a + (t_{11})a^2. The variance of the growth rate simplifies to the constant t_{11}. These results (a quadratic and a constant) would then replace those shown in Figure 1. The conditional model in (6) would simplify to only two equations, dropping the last two equations. The estimates in Table 1 would then consist only of results in the "constant" and "linear" columns. In the elaboration of the conditional model results, the equation in (7) for the effect of X_j on the status of the outcome would become a linear equation after dropping the second-order term. Thus, straight lines would replace the two quadratics for X_2 and X_5 shown in Figure 5. (The other "curves" in Figure 5 for X_1, X_3, and X_4 were already straight lines because their effects on the quadratic term were not significant and were not included in the final model. See Table 1.) Finally, the equation for the effect of X_j on the growth rate in (8) would simplify to the constant b_{1j} and the two linear functions in Figure 6 for X_2 and X_5 would become constants.

Summary

It is argued here that the typical reporting of certain HLM growth curve results at only one point in time is unnecessarily restrictive. A nonlinear growth example has demonstrated how simple matrix algebraic manipulations of standard results from HLM software can, with current technical graphics software, produce continuous descriptions of critical results over the time span of a study. To illustrate the additional information resulting from the proposed elaboration, consider the following results from the example. Given the assumed centering of time at grade 3:

- The standard deviation of the status of Reading at grade 3 is 23.4 with the standard reporting. The elaboration shown in Figure 1 indicates that the standard deviation of Reading status is relatively small at the earlier grades and then increases over time to a value of 23.4 at grade 3.
- The standard deviation of the rate of growth of Reading at grade 3, using standard reporting, is 9.57. With the elaboration, the standard deviation of the growth rate shown in Figure 2 is relatively large in the early grades, decreases somewhat for a grade or so, and then increases again to 9.57 at grade 3.
- The correlation between the status of Reading and the growth rate at grade 3 is 0.35 with standard reporting. With the elaboration, Figure 2 shows that the correlation is very strong in the early grades before decreasing to 0.35 at grade 3.

- The effects of the Xs on the status of Reading at grade 3 are shown in the first column of Table 1 using standard reporting. With the elaboration shown in Figure 5, it is seen that two of the effects (those for X_2 and X_5) are relatively high over the grade range considered, but the other three effects are very small in the early grades before increasing to values comparable with those for X_2 and X_5 at grade 3.
- The effects of the Xs on the growth rate at grade 3 using standard reporting are the results in the second column of Table 1. With the elaboration, the effects of X_1, X_3, and X_4 on the growth rate are shown in Table 6 to be constant over time. In contrast, the effects of X_2 and X_5 are estimated to be positive during the early grades but negative in the later grades.

In sum, a more complete picture of the individual growth of Reading is provided with the elaboration proposed.

Notes

1 The terms "hierarchical linear models" and "HLM" are used hereafter to refer to the general modeling approach of interest here, not to the computer software package known as HLM3.

2 Hierarchical linear modeling is used, in general, for multi-level situations with nested data. Other common applications of HLM include contextual analysis (e.g., estimation of the effects of school characteristics on the students nested in the schools) and meta-analysis (e.g., Bryk and Raudenbush, 1992).

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Author

Correspondence concerning this article should be addressed to Richard Tate, 307 Stone Building, Florida State University, Tallahassee, FL 32306 (e-mail: rtate@garnet.acns.fsu.edu).