Teaching with Multiple Strategies: An Investigation of Elementary Teachers' Understandings during a Lesson Study Cycle

Guillermo J. Farfan
Florida State University

Aki Murata
University of Florida

Alysia Roehrig
Florida State University

Abstract

Viewing teachers as learners of policy reform, this exploratory study examines a group of elementary mathematics teachers as they discussed teaching with multiple strategies as found in the new Mathematics Florida Standards during a lesson study cycle. In particular, it describes how teachers: (a) advance different explanations for teaching with multiple strategies in the new standards, and (b) anticipate or recognize major obstacles to the implementation of these new standards. Considerations of this study's results to further research on teacher professional development and educational reform are also briefly discussed.

Keywords: multiple strategies, mathematics education reform, lesson study

Introduction

In the United States, shifting policies on what constitutes suitable K–12 mathematics education often involve changes to content standards and the instructional strategies that accompany them. There is, however, a complicated relationship between the drafting and dissemination of new educational policies and the reception and subsequent implementation of said policies (Cohen & Hill, 2008; Olsen & Sexton, 2008); even so, evidence strongly indicates that teachers are key in implementing educational policies as intended (Akiba et al., 2016; Durand, Lawson, Wilcox, & Schiller, 2015; Martin, 2015). A particular issue of interest is knowing how teachers individually and collectively understand these changes, including anticipating problems that may arise when new standards are introduced (Bostic & Matney, 2013; Golding, 2017).

Currently, in the state of Florida, the mathematics content standards are the Mathematics Florida Standards (MAFS), modelled after the Common Core State Standards-Mathematics (CCSS-M; Common Core State Standards Initiative, 2016). Inheriting many of their principles from the CCSS-M, the MAFS focus mainly on mathematical ideas and practices that are both mathematically sound and foster students' problem solving and communications skills (Mathematics Florida Standards, 2014). Accordingly, attention is given in the MAFS to students being able to come up with solutions to problems using more than one strategy (i.e., with multiple

strategies), particularly in the elementary and middle grades, reflecting advances on how children develop number sense (Whitacre et al., 2011; Schoenfeld, 1992).

Incorporating multiple strategies in a lesson appears to facilitate students' mathematical reasoning and demonstration skills (Silver, Ghousseini, Gosen, Charalambous, & Strawhun, 2005; Sood & Jitendra, 2007). Rather than giving students one strategy to follow, this form of instruction can help students analyze and contrast different solutions, which in turn may highlight the underlying mathematical concept and thus facilitate students' learning process (Pesek & Kirshner, 2000; Star & Rittle-Johnson, 2008). To avoid superficial implementation of this goal, however, teachers must know how to *teach* with multiple strategies in a way that draws attention to the mathematical principles behind students' solutions. Merely allowing students to come up with different solutions is insufficient to obtain optimal results (Silver et al., 2005).

Yet, the literature suggests some teachers may have difficulties understanding and teaching in this way (Goodman, 2013; Martin, 2015; Resmovits, 2012), arguably because changes to standards and instructional policy may challenge teachers' beliefs of what mathematics is and how it should be taught to others (Hennessey, 2007). To further explore this issue, here we describe elementary mathematics teachers' responses to questions regarding the validity of teaching with multiple strategies. Specifically, we investigate the following: (a) purpose behind teaching with multiple strategies, (b) obstacles to teaching with multiple strategies, and (c) ways in which their participation in teacher professional development helped overcome challenges.

Teaching with Multiple Strategies

In our paper, teaching with multiple strategies involves allowing students to approach problems on their own and come up with, compare, and evaluate their different solutions (Jitendra et al., 2007; Lynch & Star, 2014; Silver et al., 2005). In basic arithmetic, students' ability to come up with solutions of their own happens very early. Carpenter and Moser (1984) reported that young children would solve addition and subtraction problems by using counting strategies such as "counting all" or "counting on" without any formal instruction. Importantly, students would persist using many of these strategies even after they were no longer efficient (Carpenter & Moser, 1984). Over time, research evidence revealed that teachers play a significant role in assisting students to consider other strategies that are more general and promote number sense (Carpenter et al., 1989).

Despite research done in this area, teaching young children to effectively use more than one strategy when solving arithmetic problems has remained problematic (Verschaffel, Greer, & Torbeyns, 2006). Although some of this may be due to children's overreliance on basic counting strategies to solve more complex problems (Cheng, 2012; Moeller, Martignon, Wessolowski, Engel, & Nuerk, 2011), we suggest that equally important may be that teachers are hesitant to spend valuable time teaching with multiple strategies (a time-consuming task) when standard, ready-to-use algorithms are easily available (Borko et al., 1992; Star & Rittle-Johnson, 2008), or perhaps lack the understanding of the mathematical principles behind those algorithms to recognize alternative solutions (Raveh, Koichu, Peled, & Zaslavsky, 2016).

In our case, teachers participating in lesson study (LS) professional development all wished to improve their teaching with multiple strategies in early arithmetic. The proper use of strategies such as decomposition or "make-a-ten" in developing students' number sense is well documented in the literature (Bush & Karp, 2013; Cheng, 2012; Laski, Ermakova, & Vasilyeva, 2014; Schneider & Siegler, 2010), and it is also well represented in the MAFS (e.g., MAFS.1.OA.3.6: "Use strategies such as counting on; making ten; decomposing a number leading to a ten," Mathematics Florida Standards, 2014). This strategy of "make-a-ten" became the focal point for

teachers in the LS group recorded meetings (Table 1) and provided much of the data used in this study.

Teachers as Learners in Lesson Study

Besides mathematical content, teachers also are learners of policy reform (Spillane, Reiser, & Reimer, 2002). When a new policy is implemented, teachers often gather information about said policy and integrate it with their prior knowledge, beliefs, and understandings, thereby constructing their own interpretations, meanings, and ideas of the policy (Lynch & Star, 2014; Schoenfeld, 1998). Teachers may force these ideas to fit what they know about mathematics instruction, or they may revise or reinterpret their prior knowledge in order to conform with the new policy (Dulude, Spillane, Dumay, 2015; Spillane, 2000). Furthermore, participation in professional development (PD) or learning communities might strengthen or weaken the extent to which teachers are willing to learn, adopt, and implement a new policy (Cohen, 1990; Cohen & Hill, 2001; Wood, 2007).

In this paper, LS was the medium by which teachers came to learn and discuss teaching with multiple strategies as an instructional policy. Originating in Japan, LS is a professional community model where teachers collaborate and study student learning together in pursuit of some academic goal (Gersten, Taylor, Keys, Rolfhus, & Newman-Gonchar, 2014). Although there is no one right way to "do" lesson study in Florida, all teachers in this study followed an LS cycle consisting of (a) researching and selecting a topic, (b) writing goals and develop a lesson plan, (c) teaching the research lesson, and (d) reflecting on what happened during the lesson and how it may affect future teacher practice (Gorman, Mark, & Nikula, 2010).

Ideally, LS motivates teachers to analyze and reflect on those aspects of the lesson that best help children's mathematical reasoning (Lin & Rowland, 2016), oftentimes guided by seasoned teachers who assume the role of LS facilitators, as it was the case in our sample. In fact, teachers discussed and debated with their LS facilitators a range of decisions at each stage in the LS cycle, from how to group students to what sort of manipulatives to use for the lesson. This dynamic of discussion and debate is representative of LS, where teachers rely on one another for expertise not only on content knowledge but also for other aspects of teaching and policy that may impact their classrooms (Lewis & Takahashi, 2013). Because it is entirely teacher-driven, LS provides an opportunity to observe how understandings of multiple strategies *emerged* from teachers themselves, rather than being dictated by or derived from content experts and educational researchers, as it is often the case with other PD platforms (Lewis, Perry, & Murata, 2006; cf. Lynch & Star, 2014).

Method

This exploratory study follows an embedded case study design (Yin, 2003) to collect data from elementary teachers. The first author analyzed the data following an open coding approach for qualitative data analysis (Miles, Huberman, & Saldaña, 2014; Strauss & Corbin, 1998). The first author is a former mathematics teacher and currently researches the education and professional development of mathematics teachers. The second and third authors have research expertise in lesson study and educational research, respectively, and provided feedback through discussions of results.

Setting and Participants

Data for this study were collected from three schools in Coast County District, Florida, where teachers were engaged in LS during the months of January-March of 2016.[1] The first and second authors participated in data collection at these schools as part of a larger project investigating the effectiveness and scalability of LS in Florida schools. Teachers' participation, in LS and this study, was voluntary.

In our sample (N = 11), the average teaching experience was 15.2 years; three teachers were relatively new (M = 1.3 years of teaching experience), and the rest were experienced teachers (M = 20.4 years of teaching experience). All teachers and teacher facilitators were white, female, and college educated. Four teachers, including two of the new ones, had never participated in an LS cycle before; the remainder had participated in 1 to 13 previous LS cycles (Table 1).

Table 1. *Teachers' Background Information*

Name	Role	School	Experience*	LS Cycles	Type of Data
Barbara	Teacher	Journeys	11	0	Video
Carol	Teacher	Journeys	2	0	Video
Lise	Teacher	Journeys	14	1	Video
Marie	Facilitator	Journeys	24	13	Video/interview
Rosalind	Teacher	Journeys	20	0	Video
Ada	Teacher	Plantation	28	1	Interview
Irene	Facilitator	Trademark	11	6	Interview
Jocelyn	Teacher	Trademark	1	2	Interview
Martha	Teacher	Plantation	1	0	Interview
Rachel	Facilitator	Plantation	26	10	Interview
Rita	Facilitator	Trademark	29	7	Interview

* Total number of years teaching.

Data Sources

Data for this study came from two sources: video recordings of meetings by one LS group at one school (Journeys), and personal interviews with teachers and LS facilitators from two other schools (Plantations and Trademark) at the end of the LS cycle.

Video data. We only had access to the recordings of the LS group that met at the start of spring 2016 (at Journeys) for analysis. The LS group consisted of four first-grade teachers and one LS facilitator who met five times, for half a day or more, over a two-month period to work on a research lesson. Teachers recorded videos of these five meetings themselves with equipment provided by the research team, which they returned upon completion of the LS cycle. Altogether, these meetings amounted to 9 hours of video.

The focus of the LS group research lesson was decomposing and adding two-digit numbers as per MAFS.1.NBT.3.4 (Mathematics Florida Standards, 2014). The LS group divided the lesson into three tasks: a simple task of "Solve 53 + 4," a more complex task of "Solve 57 + 6" (which requires using the "make-a-ten"), and a more challenging activity where students will add two-

[1] To preserve confidentiality, all names of schools, participants, and districts in this paper are pseudonyms.

digit numbers using a videogame (Minecraft). More importantly, teachers in this LS group expected students to be able to see the similarities underlying their different strategies and connect them back to the idea of "make-a-ten."

Interview data. In an effort to obtain a wider range of perspectives beyond one LS group's meetings, we also interviewed additional teachers (n =3) and LS facilitators (n =3) from two other schools at the end of their LS cycles in spring 2016. The LS facilitator for the video recorded LS group also provided an interview. The interviews focused on teachers' experience with LS in addition to questions regarding the new standards and their teaching practice (e.g., "What did you learn about collaborating with other teachers through this lesson study?", "How would you say your experience and knowledge of mathematics [after LS cycle] have prepare you for these changes [in standards]?"). In all, there were just over 4 hours of audio recorded interviews, with the average interview lasting 35 minutes.

Data Analysis

Video and interview data were first transcribed word-for-word and later open coded using Microsoft Word and Excel. In the first cycle of coding, the first author coded everything first, then grouped codes related to the research questions, leaving out unrelated ideas (e.g., classroom management, use of manipulatives). In the second cycle of coding, the first author went back to our codes and bracketed whole paragraphs (constituting an "instance") into categories based on whenever a statement regarding multiple strategies' purpose or their necessity has been made, whenever a major obstacle to teaching with multiple strategies or the MAFS has been identified, or whenever teachers elaborated on how they benefited from LS or other PDs in light of the new standards. This process continued until data saturation was reached (i.e., no more instances were found). Resulting codes and categories are shown in Table 2.

Results

Understanding Multiple Strategies in the MAFS

Teachers offered a number of different explanations of what multiple strategies stand for and what they can accomplish. For instance, when teachers discussed why multiple strategies should be part of mathematics instruction, the most common response was that their inclusion is meant to foment students' *conceptual understanding*. There are 17 instances of this explanation in the combined data, 12 of which occurred during LS group discussions alone. Interestingly, the least popular explanations were that teaching with multiple strategies is meant to provide students with a *skillset* needed for the higher grades, and that they help improve students' *arithmetic proficiency* (Table 2).

A closer look at these instances revealed that the most elaborate explanations occurred during the LS group planned meetings. In these meetings, Marie (the LS group facilitator) often exercised a notable influence as she highlighted points for discussion and redirected teachers' attention to aspects of the lesson that may have gone unnoticed. For example, early in the LS cycle, Marie asked her fellow teachers why they should use multiple strategies in their lesson:

> Marie: I do think, because we keep talking about different strategies, and I think it's important to get clear on why… why use different strategies? So… if we're teaching children how to add, why do make a ten? "5 + 8 = I'll take a 5 and make 10 plus 3 more," the make-a-ten strategy. Why don't, "5 + 8 = [counting fingers] 9, 10, 11, 12, 13." Why give them different strategies? So, what's the purpose in it?
>
> Carol: Different types of learners.

Marie: Ok, so we have different entry points because we've different types of learners.

Barbara: Also, because some of the strategies built upon each other. You've to learn the basic strategy for addition or decomposing a number before they can add double-digit numbers in their minds, or their heads.

Marie's following explanation is rather different than Carol's or Barbara's:

Marie: And this is important to me. I do different strategies to build number sense. So, it's really building their number sense. Going back to 25 + 8, doing that make-a-ten, they know that 30 is nearby. It's not just "let's do the trick: making a ten!" and everybody has to do this neat trick of making a ten. It's reasoning, it's sitting there and thinking "Ok. I've a 25. What friendly number is nearby?" kind of thing… By using different strategies, it promotes discussion. So, when we're adding 25 + 26, I can take 5 from the 26 and make it 30. And then add 21 to it. Or I might think of 25 + 25, and then add 1 to it—that discussion promotes number sense.

Table 2. *Coding Structure and Number of Instances per Category*

Category	Codes	Sample	# Instances
Multiple Strategies	Conceptual understanding	"I think that the focus is on students really getting the conceptual understanding…understanding why."	17 (12)
	Verbal reasoning/ explanation	"I like that they don't want to just solve the problem, but really explain and analyze what they're doing."	14 (7)
	Accommodate different learners	"[…] their favorite part is when you learn all these strategies, and I said, 'Well, what strategy would you like to use?' That's their favorite. That's usually when they take it on their own."	9 (4)
	Exploration/ collaboration	"[…] having all these different strategies, students can help each other figure out what works best for them."	8 (3)
	Arithmetic proficiency	"Also, because some of the strategies built upon each other. You've to learn the basic strategy for addition, or decomposing a number, before the can add double-digit numbers."	5 (4)
	Skillset	"As long as they're building later on…find what works for them. And then, kinda built upon that."	2 (2)
Obstacles	Old epistemic beliefs	"It's a challenge for some teachers because we're used to the old algorithms, and just teaching them in that one way."	15 (8)
	Inconsistent support	"I don't know if it's just our district, or if it's statewide, but we don't have the curriculum or the materials to back it up."	7 (2)
	Developmentally inappropriate	"And some of them mentally are not ready for the strategy that their asking […] They're not."	6 (6)
LS benefits	Meet challenges	"[…] that's something you do in lesson study, as you're writing the lesson, you come up with every possible way a student can solve this problem and think about what the teacher's reaction would be."	7 (2)
	Fill in gaps	"And my [LS] team has helped...talked about what needs to happen, and in what order, so we do talk about that."	1 (0)
	No change	"I don't think it has been much of a change. I think it has been more of the explanation piece that it's probably the biggest change."	1 (0)

Note. Numbers in parentheses ( ) indicate those instances, included in the total, that occurred during LS planned meetings discussions.

Marie appears to differentiate between knowing to compose/decompose the number 10 (a "neat trick") and strategizing a solution ("sitting there and thinking") that reflects a good understanding of numbers (Jitendra et al., 2007; Schoenfeld, 1992; Whitacre et al., 2011). For Marie, the ultimate reason for teaching with multiple strategies lies in their ability to encourage class discussion and student reasoning, which in turn leads to a grasp of the concept of number.

This close association between multiple strategies and developing students' number sense only occurred in our data from the LS group's discussions, led by Marie. Even so, not everyone shared Marie's detailed explanation; Carol, at another point, said her biggest issue with using multiple strategies was that "it doesn't matter how they (students) got the answer…if they can figure out what 2 + 2 is, on whatever strategy they use, and they get the right answer, *what's the problem?*" (emphasis added). This sort of response is contrary not only to Marie's but to the reform-based principles behind the MAFS (Mathematics Florida Standards, 2014; see also Coburn, Hill, & Spillane, 2016; Schoenfeld, 2014). Carol's argument, however, is not entirely novel, as similar objections from teachers exist in the literature (Bingolbali, 2011; Silver et al., 2005).

Although no detailed references to students' number sense appeared in the personal interviews, teachers did mention other conceptual advantages of using multiple strategies, often tied to students' ability to explain their solutions, as when a new teacher elucidated:

> Jocelyn: My lower students, who don't understand [the standard algorithms], are using all these different strategies now, and they're doing a good job at explaining what they know because they understand the process, whereas some of the high students, using the old algorithms, just know that you crossed out the zero, and it magically becomes ten, because they crossed something else out. They're actually struggling. It's really interesting.

Experienced teachers also highlighted the importance of students providing explanations for their chosen strategies both during group meetings (the "whole point of Common Core[2] [is] for students to explain [the solution] so that somebody else understands"—Rosalind), and personal interviews ("to have an understanding of math that they [students] can explain… if you can tell to somebody what you've done, you understand what you're doing"—Rachel). Even though these responses lacked the context of shared discussion provided in LS planned meetings, they nonetheless show teachers' awareness of some of the conceptual aspects involved in teaching with multiple strategies.

We note also that teachers' understandings of multiple strategies were not always uniform or consistent, even among LS facilitators. For instance, Rachel (another LS facilitator) who highlighted the importance of students being able to explain their strategies, also appeared to endorse the idea of using multiple strategies to support different types of learners:

> Rachel: All of those kids can do math; they just didn't do it the same way. They all got their answers, they all were a success. They all looked like they felt good about themselves. To me, that's a successful lesson.

This is somewhat different from Marie's idea of a successful lesson:

> Marie: These were first graders. They are six years old. And they were talking math extremely well. They were very comfortable in trying to explain their ideas. Very few of them just did [answer]: "Oh, I just did it. It just happened." They were reaching [out] to communicate their ideas…This

[2] In interviews and planned meetings, teachers frequently spoke of the MAFS as "Common Core" or otherwise as the "new standards," but never by their official name. Teachers' original wording has been kept whenever direct excerpts or quotes are used.

> lesson brought out their ability to communicate and probably helped develop it in some way.

Altogether, this range of responses suggests that teachers' explanations of what multiple strategies stand for and what they can accomplish are not monolithic or uniform. In some cases, they contradict each other, as with Marie's and Carol's views regarding students' number sense. In other cases, such as with Rachel, teachers may entertain more than one view. Given that teaching with multiple strategies can fulfill more than one instructional goal (Silver et al., 2005), this diversity of views is not surprising.

Obstacles to Teaching with Multiple Strategies

One of the obstacles identified were *epistemic beliefs,* which are beliefs and ideas about what mathematical knowledge is and how it should be taught to others (Depaepe, De Corte, & Verschaffel, 2016; Schoenfeld, 2014). Teachers spoke of these beliefs as a major obstacle against teaching with multiple strategies (15 instances). Often, teachers highlighted these old ideas as holding people back from embracing the new standards. For instance, Lise raised the following issue about parents during an LS group meeting:

> Lise: And these ridiculous strategies that are being tested, "well, you know, we're going to test for this strategy, and that's why we're going to do it." That's why all these parents are freaking out, "Why would I use *that* strategy when I can solve the problem *like this*?" (emphasis added)

Other teacher colleagues also failed to see the point of teaching with multiple strategies, as the LS facilitator Rita highlighted during her interview that upper grades teachers "are all up in arms," remarking to her "'Why should I do that, when I can just teach them to align their place values [like this]?'" Other times, teachers in LS compared teaching with multiple strategies against procedural instruction; for instance, Irene expressed how for many of her fellow teachers, teaching formulas and algorithms mechanically "is their comfort zone, so that's what they teach, so the students never learn to think through the problem."

As shown in Table 2, apart from conceptually challenging parents' and colleagues' beliefs, another major obstacle to teaching with multiple strategies was *inconsistent support* from state and district leaders. Tellingly, the overwhelming majority of teachers who mentioned this were experienced teachers. For instance, Rita complained how one year they "were taught [by the district to do] one thing, and the next, it's kinda a whole different [thing]," while Irene said her district was not "providing the curriculum necessary" to meet the standards "in an orderly way" (both Rita and Irene were LS facilitators).

We mentioned earlier that teachers may be disinclined to spend valuable time learning to teach with multiple strategies when other alternatives, such as teaching the standard algorithm, are readily available. Without proper support, this can lead to even more frustration ("we're all inundated, just all the time, so much to think about. And you're going to ask me to think about just one more thing?"—Rita), or even denial ("some teachers are feeling: 'well I was not taught this way, you aren't providing training, so I'm gonna do it the way I've always taught.'"—Marie).

Notably, new teachers never mentioned lack of district support as troublesome in their interviews. Instead, they highlighted only the persistence of old teaching strategies (Jocelyn), or had reservations regarding parents, who may not know anything beyond the old algorithms (Martha and Carol). Although our sample had considerably more experienced teachers (n = 8) than new teachers (n = 3), the fact that new teachers made no mention of district leadership or teacher training as a source of strain makes it an intriguing finding.

Finally, an obstacle that came up in the planned meetings, but did not surface during the interviews, is that teaching with multiple strategies may be *developmentally inappropriate* to at least a subgroup of children. However, rather than being theoretically motivated, this objection was always made with particular children in mind, as when Rosalind mentioned some of her students "might not be ready to do all 5 strategies," or when Carol said some children in her class are "terrified" of having to come up with more than one solution (see Lynch & Star, 2014, where a similar finding is discussed; Clements, Fuson, & Sarama, 2017).

Lesson Study Benefits

We noted seven instances where teachers mentioned their participation in LS and how that helped *meet challenges* of teaching with multiple strategies; for example, being part of an LS cycle allowed one to share "about these perspectives, understandings, Common Core, and the standards, and how to teach them." LS and Thinking Math (another instructional PD) "opened" a teacher facilitator's mind to the advantages of teaching with multiple strategies, putting her (Rachel) "ahead of the game" as it were. In another case, going through LS helped another teacher facilitator empathize with her students, by making her receptive to how students go through and learn these strategies on their own ("I think that also helps me, because then I can sort of get the idea, 'Ok, now I know what they [the students] don't get. Now, I know what *you* don't get.'"—Rita). And, at least in one case, participation in LS appears to have assisted teachers filling-in for the gaps in district support (*fill in gaps*; "And my [LS] team has helped. And the first-grade team sat down together, you know, and talked about what needs to happen, and in what order, so we do talk about that" –Ada).

On at least two occasions, teachers in the LS group also spoke of the benefits of LS during their planning meetings. This happened most vividly when teachers were trying to come up with meaningful tasks to elicit different student strategies. In the following, Barbara and the others have been trying to "Solve 53 + 4" and "Solve 57 + 6" using unit blocks for a few minutes, attempting to anticipate what students might come up with when using "make-a-ten":

> Barbara: That's how in their heads, how they'll do it [points to a table]. I don't know how they're going to put it on paper, but that's how they're going to explain to me. I would do it this way—the easy way.
>
> Marie: Well, that would be good. To see our ways, and then you kinda compare. And that's something you do in lesson study, as you're writing the lesson, you come up with every possible way a student can solve this problem and think about what the teacher's reaction would be.

Once again, it was Marie who directed teachers' attention to students' mathematical thinking, in keeping with her role as a LS facilitator (Gorman, Mark, & Nikula, 2010; Lewis, Perry, & Murata, 2006).

Discussion

The overall impression that emerges from both interview and LS group meeting data is that teachers primarily find teaching with multiple strategies conceptually useful, albeit not always for the same reasons. Of these, the idea that teaching with multiple strategies should support the development of students' number sense is the explanation that comes closest to the principles behind the MAFS and CCSS-M (Clements, Fuson, & Sarama, 2017; Schoenfeld, 2014; Zimba, 2016). The idea that doing so helps develop students' verbal reasoning is a close second. Arguably, these two explanations may reflect elementary teachers' comparative familiarity with reform-based type instruction in the United States (Chestnut, 2017, pp. 85–89; Granthum, 2016, p. 75; Marbach-Ad & McGinnis, 2010; cf. Lynch & Star, 2014).

However, there were other teacher explanations about teaching with multiple strategies that were more difficult to interpret. For instance, the idea of using multiple strategies to accommodate different types of learners could be interpreted as attending to students' needs along their learning progressions or trajectories (Empson, 2011; Lobato & Walters, 2017). Or it could be interpreted as resembling the controversial idea of "learning styles," as some have argued in the past (Lynch & Star, 2014; Pashler, McDaniel, Rohrer, & Bjork, 2008). If teachers are mixing what they know about multiple strategies with ideas found elsewhere, then more needs to be done to clarify how that may interfere with appropriate policy implementation (Spillane, 2000; Silver, Ghousseini, Gosen, Charalambous, & Strawhun, 2005).

Even so, we believe it is a good sign that teachers in LS were not blindly accepting a top-down definition (e.g., coming from district leaders) of what teaching with multiple strategies is or not. Rather, the opportunity to discuss, compare, and critique different ideas about what teaching with multiple strategies entails may actually have increased the chances of correct implementation and buy-in (Dulude, Spillane, & Dumay, 2011).

Elementary teachers in this study also identified two main sources of strain or obstacles to teaching with multiple strategies as a new policy: (a) using multiple strategies challenges parents' and teachers' prior beliefs, and (b) inconsistent support from state/district leadership when it comes to school and classroom implementation. There is no denying that teaching with multiple strategies as per the CCSS-M represents a more rigorous approach to mathematics instruction, and there has been widespread mistrust and misunderstanding in the United States regarding how these new changes in instructional policy are meant to be realized (Wu, 2011; Zimba, 2016). Our results showed that there are teachers who share some of the same objections that the general public may have regarding using multiple strategies to teach mathematics to children, such as not being developmentally appropriate or not being necessary so long as the answer is correct (Clements, Fuson, & Sarama, 2017). Note, however, that in our data neither experienced nor new teachers outright *opposed* teaching with multiple strategies or the MAFS, even if there were some who had reservations.

On the other hand, we did find instances where teachers expressed dissatisfaction with the district and state leadership. In this case, there was a subtle but significant difference between experienced and new teachers' responses during interviews. Although we abstain from speculating at length without more data, we believe a plausible explanation is that teachers are accessing different frames of reference when answering these questions (Spillane, Reiser, & Reimer, 2002). Conceivably, experienced teachers' prior experience with policy implementation, which new teachers did not have, may have played a role in their meaning-construction of the new policy (Lynch & Star, 2014; Spillane, 2000). Alternatively, new teachers may have shared the same sentiment as experienced teachers but chose to not voice their criticisms publicly. Either way, the fact that there is dissatisfaction about the role of district and state leadership should be a sign of concern. If indeed teachers are key in implementing reform-based policies as intended (Martin, 2015), then districts should pay more attention to what their teachers are thinking and saying—particularly experienced teachers, who might have to change the most whenever new standards are introduced (Schoenfeld, 2014).

Finally, we found some limited but encouraging evidence that participating in LS has helped some teachers in their learning of what teaching with multiple strategies entails (e.g., during the LS group discussions led by Marie). Identifying those PD efforts that support teacher learning opportunities and focused on students' sense-making may be another way to ensure teachers learn to implement reform-based policies in the intended way (Borko, 2004; Bostic & Matney, 2013).

Limitations

Given its exploratory nature, there are a number of limitations to this study. First, participation in LS and the project was voluntary, meaning teachers in this study were self-motivated to become part of LS. It is conceivable, then, that they were also more self-motivated to learn about multiple strategies and the MAFS than their non-participant peers. Although we widened the net of participating teachers to include people from more than one school in the district, their views and ideas expressed here might still not be representative of the teacher population in Florida as a whole.

Second, in our sample there were more experienced teachers (measured by years of teaching experience) than new teachers, though most experienced teachers were also relatively new to LS. This was not by design; it is quite possible that new teachers, still unfamiliar with their many responsibilities and trying to make a good impression (teachers are no longer tenured in the state of Florida), will be more hesitant to make the time commitment necessary, including time away from their classrooms, to participate in LS.

Conclusion

We hope these preliminary findings help generate discussions about how to help teachers educate and train themselves in teaching with multiple strategies. The promotion of standards such as the CCSS-M in the United States requires that teachers teach many mathematical topics in an unfamiliar way (Wilson & Downs, 2014). Our results suggest that, although most teachers seemed willing to teach with multiple strategies, their views on what teaching said strategies entail, or what they were set to accomplish, were not always uniform or consistent.

When looking at the big picture, it is helpful to recognize that teachers are also *learners* of policy reform and we cannot expect that they will know how to adopt and interpret new instructional policies merely by making information about them available. Doing so ignores conceptual and pedagogical challenges new policies may represent, as well as teachers' interactions with colleagues, their school and district leadership, and their prior experiences with PD. Rather than simply requiring teachers to "know the standards" or "learn more math," district and school leaders would do well in identifying PD, such as LS, that provides a forum for teachers to learn how to implement new standards in a way that foments, not hinders, student learning.

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Corresponding Author: Guillermo J. Farfan
Author Contact Information: gjf14@my.fsu.edu

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